Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (B) $a + (n-1)d$

Reason: The standard formula for the n-th term of an AP with first term a and common difference d is:

$$T_n = a + (n-1)d$$

Q2. Correct Option: (C) $a \cdot r^{n-1}$

Reason: The standard formula for the n-th term of a GP with first term a and common ratio r is:

$$T_n = a \cdot r^{n-1}$$

Q3. Correct Option: (C) 59

Reason: The AP is $3,\ 7,\ 11,\ 15, \ldots$ so $a = 3$, $d = 4$.

$$T_{15} = a + (15-1)d = 3 + 14 \times 4 = 3 + 56 = 59$$

Q4. Correct Option: (B) 670

Reason: The AP is $5,\ 8,\ 11, \ldots$ so $a = 5$, $d = 3$, $n = 20$.

$$\begin{aligned} S_{20} &= \frac{n}{2}\big[2a + (n-1)d\big] \\ &= \frac{20}{2}\big[2(5) + 19(3)\big] \\ &= 10\big[10 + 57\big] = 10 \times 67 = 670 \end{aligned}$$

Q5. Correct Option: (B) 242

Reason: The GP is $2,\ 6,\ 18, \ldots$ so $a = 2$, $r = 3$, $n = 5$.

$$S_5 = a \cdot \frac{r^n - 1}{r - 1} = 2 \cdot \frac{3^5 - 1}{3 - 1} = 2 \cdot \frac{243 - 1}{2} = 2 \cdot \frac{242}{2} = 242$$

Q6. Correct Option: (B) $1,\ 3,\ 9,\ 27, \ldots$

Reason: A GP requires a constant ratio between successive terms.

$$\frac{3}{1} = \frac{9}{3} = \frac{27}{9} = 3 \quad \text{(constant common ratio } r = 3\text{)}$$

Options (A) and (C) are APs (constant difference, not ratio). Option (D) has differences $3, 4, 5, \ldots$ (neither AP nor GP).

Q7. (A): Both A and R are true, and R is the correct explanation of A.

Reason:

- The sequence $128,\ 64,\ 32,\ 16, \ldots$ is a GP with $a = 128$ and $r = \frac{64}{128} = \frac{1}{2}$. Since $|r| = \frac{1}{2} < 1$, the sum to infinity exists.

- Using the formula stated in R:

$$S_\infty = \frac{a}{1-r} = \frac{128}{1-\frac{1}{2}} = \frac{128}{\frac{1}{2}} = 256$$

- **Assertion A is TRUE. Reason R is TRUE** and it directly provides the formula used to establish A. Hence R is the correct explanation of A.

Q8. Correct Option: (B) 8

Reason: Let the three consecutive terms of the AP be $a-d,\ a,\ a+d$.

Using the sum condition:

$$\begin{aligned}(a-d)+a+(a+d) &= 24\\ 3a &= 24\\ a &= 8\end{aligned}$$

(The product condition = 480 can be used to find d, but a is determined solely from the sum.)

Q9. Correct Option: (C) $\frac{2}{3}$ **or** $\frac{3}{2}$

Reason: Let the three numbers in GP be $\frac{a}{r},\ a,\ ar$.

Use the product condition:

$$\begin{aligned}\frac{a}{r}\cdot a\cdot ar &= 216\\ a^3 &= 216\\ a &= 6\end{aligned}$$

Use the sum condition:

$$\begin{aligned}\frac{6}{r}+6+6r &= 19\\ \frac{6}{r}+6r &= 13\end{aligned}$$

Multiplying throughout by r:

$$6r^2-13r+6=0$$

Solve the quadratic:

$$r = \frac{13 \pm \sqrt{169-144}}{12} = \frac{13 \pm \sqrt{25}}{12} = \frac{13 \pm 5}{12}$$

$$r = \frac{18}{12} = \frac{3}{2} \qquad \text{or} \qquad r = \frac{8}{12} = \frac{2}{3}$$

Q10. Correct Option: (C) 6

Reason: Given $S_n = 3n^2 + 2n$.

Find the general term T_n:

$$\begin{aligned} T_n &= S_n - S_{n-1} \\ &= \big(3n^2 + 2n\big) - \big(3(n-1)^2 + 2(n-1)\big) \\ &= 3n^2 + 2n - 3(n^2 - 2n + 1) - 2n + 2 \\ &= 3n^2 + 2n - 3n^2 + 6n - 3 - 2n + 2 \\ &= 6n - 1 \end{aligned}$$

Find the common difference:

$$\begin{aligned} T_1 &= 6(1) - 1 = 5 \\ T_2 &= 6(2) - 1 = 11 \\ d &= T_2 - T_1 = 11 - 5 = 6 \end{aligned}$$

(Since $T_n = 6n - 1$ is linear in n, the common difference is the coefficient of n, which is **6**.)

Q11. Correct Option: (A) 0

Reason: Let the AP have first term $a = 2$ and common difference d.

$$\begin{aligned} T_4 &= 2 + 3d \\ T_7 &= 2 + 6d \\ T_{10} &= 2 + 9d \end{aligned}$$

For $T_4,\ T_7,\ T_{10}$ to be in GP, the condition is $T_7^2 = T_4 \cdot T_{10}$:

$$\begin{aligned} (2 + 6d)^2 &= (2 + 3d)(2 + 9d) \\ 4 + 24d + 36d^2 &= 4 + 18d + 6d + 27d^2 \\ 4 + 24d + 36d^2 &= 4 + 24d + 27d^2 \\ 36d^2 &= 27d^2 \\ 9d^2 &= 0 \\ d &= 0 \end{aligned}$$

Hence $d = 0$. (When $d = 0$, all three terms equal 2, forming a GP with $r = 1$.)

Section B — Short Answer Questions

Q12. Given: $T_n = 5n - 3$.

(i) **First three terms and common difference:**

$$\begin{aligned} T_1 &= 5(1) - 3 = 2 \\ T_2 &= 5(2) - 3 = 7 \\ T_3 &= 5(3) - 3 = 12 \end{aligned}$$

The first three terms are **2**, **7**, **12**.
Common difference:

$$d = T_2 - T_1 = 7 - 2 = \mathbf{5}$$

(Alternatively, since $T_n = 5n - 3$ is linear in n, the coefficient of n directly gives $d = 5$.)

(ii) **Finding n such that $T_n = 97$:**

$$\begin{aligned} 5n - 3 &= 97 \\ 5n &= 100 \\ n &= \mathbf{20} \end{aligned}$$

Hence the 20th term of the AP equals 97.

Q13. Identify the first and last terms:

We need integers from 100 to 300 (inclusive) divisible by 7.

$$\begin{aligned} \text{First term:} \quad a &= 105 \quad (7 \times 15 = 105,\ \text{since } 7 \times 14 = 98 < 100) \\ \text{Last term:} \quad l &= 294 \quad (7 \times 42 = 294,\ \text{since } 7 \times 43 = 301 > 300) \end{aligned}$$

Common difference: $d = 7$.

Find the number of terms:

$$\begin{aligned} T_n &= a + (n-1)d \\ 294 &= 105 + (n-1) \times 7 \\ 189 &= (n-1) \times 7 \\ n - 1 &= 27 \\ n &= 28 \end{aligned}$$

There are **28** such terms.

Compute the sum:

$$S_{28} = \frac{n}{2}(a + l) = \frac{28}{2}(105 + 294) = 14 \times 399 = \mathbf{5586}$$

Q14. Let the first term be a and common ratio be r.

$$T_3 = ar^2 = \frac{1}{2} \quad \cdots (1)$$

$$T_6 = ar^5 = \frac{1}{16} \quad \cdots (2)$$

Find r by dividing equation (2) by equation (1):

$$\frac{ar^5}{ar^2} = \frac{\frac{1}{16}}{\frac{1}{2}}$$

$$r^3 = \frac{1}{16} \times \frac{2}{1} = \frac{1}{8}$$

$$r = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

Find a using equation (1):

$$a \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$
$$a \cdot \frac{1}{4} = \frac{1}{2}$$
$$a = \frac{1}{2} \times 4 = 2$$

Write the first five terms of the GP:

$$a = 2, \quad r = \frac{1}{2}$$

$$T_1 = 2, \quad T_2 = 2 \times \frac{1}{2} = 1, \quad T_3 = 1 \times \frac{1}{2} = \frac{1}{2}, \quad T_4 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}, \quad T_5 = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$\text{GP:} \quad 2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8}$$

Q15. The GP is $3,\ 6,\ 12,\ 24, \ldots$ with $a = 3$, $r = 2$.

Write the formula for S_n:

$$S_n = a \cdot \frac{r^n - 1}{r - 1} = 3 \cdot \frac{2^n - 1}{2 - 1} = 3(2^n - 1)$$

Set up the inequality $S_n > 3000$:

$$3(2^n - 1) > 3000$$
$$2^n - 1 > 1000$$
$$2^n > 1001$$

Find the least n by testing powers of 2:

$$2^9 = 512 < 1001$$
$$2^{10} = 1024 > 1001 \quad \checkmark$$

Verification:

$$S_{10} = 3(2^{10} - 1) = 3(1024 - 1) = 3 \times 1023 = 3069 > 3000 \quad \checkmark$$
$$S_9 = 3(2^9 - 1) = 3(512 - 1) = 3 \times 511 = 1533 < 3000$$

The **least number of terms** required is $\mathbf{n = 10}$.

Q16. We need to insert 4 arithmetic means between 3 and 23, giving a total of 6 terms:

$$3,\ A_1,\ A_2,\ A_3,\ A_4,\ 23$$

Find the common difference d:

Here $a = 3$, $T_6 = 23$, $n = 6$.

$$\begin{aligned} T_6 &= a + (6-1)d \\ 23 &= 3 + 5d \\ 5d &= 20 \\ d &= 4 \end{aligned}$$

Find the four arithmetic means:

$$\begin{aligned} A_1 &= 3 + 4 = 7 \\ A_2 &= 3 + 8 = 11 \\ A_3 &= 3 + 12 = 15 \\ A_4 &= 3 + 16 = 19 \end{aligned}$$

The complete sequence is: **3, 7, 11, 15, 19, 23**.

Verification:

$$\begin{aligned} 7 - 3 &= 4 \\ 11 - 7 &= 4 \\ 15 - 11 &= 4 \\ 19 - 15 &= 4 \\ 23 - 19 &= 4 \end{aligned}$$

The common difference between every pair of consecutive terms is 4 (constant). Hence the sequence is a valid AP. ✓

Section C — Long Answer Questions

Q17. Given: First step requires 50 bricks; each subsequent step requires 4 more bricks. Total steps = 15.

(i) **Sequence and verification:**
The number of bricks per step:

$$50,\ 54,\ 58,\ 62,\ \ldots$$

$$54 - 50 = 4, \quad 58 - 54 = 4, \quad 62 - 58 = 4$$

The difference between consecutive terms is constant. Hence the sequence is an **AP** with:

$$a = 50, \qquad d = 4$$

(ii) **Bricks used in the 10th step:**

$$T_{10} = a + (10-1)d = 50 + 9 \times 4 = 50 + 36 = \mathbf{86}$$

(iii) **Total bricks for the entire staircase ($n = 15$):**

$$\begin{aligned} S_{15} &= \frac{n}{2}\big[2a + (n-1)d\big] \\ &= \frac{15}{2}\big[2(50) + 14(4)\big] \\ &= \frac{15}{2}\big[100 + 56\big] \\ &= \frac{15}{2} \times 156 = 15 \times 78 = \mathbf{1170} \text{ bricks} \end{aligned}$$

(iv) **Maximum steps within a budget of 1500 bricks:**
We require $S_n \leq 1500$:

$$\begin{aligned} \frac{n}{2}\big[2(50) + (n-1)(4)\big] &\leq 1500 \\ \frac{n}{2}\big[96 + 4n\big] &\leq 1500 \\ n(48 + 2n) &\leq 1500 \\ 2n^2 + 48n - 1500 &\leq 0 \\ n^2 + 24n - 750 &\leq 0 \end{aligned}$$

Solving $n^2 + 24n - 750 = 0$:

$$n = \frac{-24 + \sqrt{576 + 3000}}{2} = \frac{-24 + \sqrt{3576}}{2} \approx \frac{-24 + 59.8}{2} \approx 17.9$$

So $n \leq 17$. Since the staircase has only 15 steps and $S_{15} = 1170 \leq 1500$, the contractor can build **all 15 steps** within the budget, using 1170 bricks with 330 bricks to spare.

Q18. Ball dropped from 48 m; after each bounce it rises to $\dfrac{2}{3}$ of the previous height.

(i) **Heights after the first four bounces and verification:**

$$\begin{aligned} \text{After 1st bounce:} &\quad 48 \times \tfrac{2}{3} = 32 \text{ m} \\ \text{After 2nd bounce:} &\quad 32 \times \tfrac{2}{3} = \tfrac{64}{3} \text{ m} \\ \text{After 3rd bounce:} &\quad \tfrac{64}{3} \times \tfrac{2}{3} = \tfrac{128}{9} \text{ m} \\ \text{After 4th bounce:} &\quad \tfrac{128}{9} \times \tfrac{2}{3} = \tfrac{256}{27} \text{ m} \end{aligned}$$

Ratios between consecutive heights:

$$\frac{\frac{64}{3}}{32} = \frac{2}{3}, \qquad \frac{\frac{128}{9}}{\frac{64}{3}} = \frac{2}{3}, \qquad \frac{\frac{256}{27}}{\frac{128}{9}} = \frac{2}{3}$$

The ratio is constant. Hence the sequence is a **GP** with:

$$a = 32 \text{ m}, \qquad r = \frac{2}{3}$$

(ii) **Height after the 6th bounce:**

$$T_6 = a \cdot r^5 = 32 \times \left(\frac{2}{3}\right)^5 = 32 \times \frac{32}{243} = \frac{1024}{243} \approx \mathbf{4.21} \text{ m}$$

(iii) **Total distance until the ball hits the ground for the 5th time:**
Counting impacts:

- 1st hit: initial drop of 48 m.
- 2nd hit: rises 32 m, falls 32 m.
- 3rd hit: rises $\frac{64}{3}$ m, falls $\frac{64}{3}$ m.
- 4th hit: rises $\frac{128}{9}$ m, falls $\frac{128}{9}$ m.
- 5th hit: rises $\frac{256}{27}$ m, falls $\frac{256}{27}$ m.

$$\begin{aligned}\text{Total} &= 48 + 2\left(32 + \frac{64}{3} + \frac{128}{9} + \frac{256}{27}\right)\\ &= 48 + 2 \times 32\left(1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27}\right)\end{aligned}$$

Sum of the GP in the bracket ($a = 1$, $r = \frac{2}{3}$, $n = 4$):

$$1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} = \frac{27 + 18 + 12 + 8}{27} = \frac{65}{27}$$

$$\text{Total} = 48 + 64 \times \frac{65}{27} = \frac{1296}{27} + \frac{4160}{27} = \frac{5456}{27} \approx \mathbf{201.9} \text{ m}$$

(iv) **After which bounce does height first fall below 3 m?**
We need $T_n = 32 \times \left(\frac{2}{3}\right)^{n-1} < 3$.
Testing successive values:

$$\begin{aligned}&T_5 = 32 \times \left(\tfrac{2}{3}\right)^4 = 32 \times \tfrac{16}{81} = \tfrac{512}{81} \approx 6.32 \text{ m} > 3\\ &T_6 = \tfrac{1024}{243} \approx 4.21 \text{ m} > 3\\ &T_7 = 32 \times \left(\tfrac{2}{3}\right)^6 = 32 \times \tfrac{64}{729} = \tfrac{2048}{729} \approx 2.81 \text{ m} < 3 \quad \checkmark\end{aligned}$$

The height first falls below 3 m **after the 7th bounce**.

Q19. Plan A (AP): $a = 500$, $d = 100$. **Plan B (GP):** $a = 500$, $r = 1.2$.

(i) **Plan A deposit in the 12th month:**

$$T_{12} = 500 + (12 - 1) \times 100 = 500 + 1100 = \textbf{Rs 1600}$$

(ii) **Plan A total savings over 12 months:**

$$S_{12} = \frac{12}{2}\big[2(500) + 11(100)\big] = 6\big[1000 + 1100\big] = 6 \times 2100 = \textbf{Rs 12,600}$$

(iii) **Plan B deposit in the 6th month:**

$$T_6 = 500 \times (1.2)^5 = 500 \times 2.488 = \textbf{Rs} \ \ \mathbf{1244}$$

(iv) **Comparison after 10 months:**

Plan A — Total over 10 months:

$$S_{10}^{A} = \frac{10}{2}\big[2(500) + 9(100)\big] = 5\big[1000 + 900\big] = 5 \times 1900 = \textbf{Rs} \ \ \mathbf{9{,}500}$$

Plan B — Total over 10 months:

$$S_{10}^{B} = 500 \times \frac{(1.2)^{10} - 1}{1.2 - 1} = 500 \times \frac{6.192 - 1}{0.2} = 500 \times \frac{5.192}{0.2} = 500 \times 25.96 = \textbf{Rs} \ \ \mathbf{12{,}980}$$

Conclusion: Since Rs 12,980 > Rs 9,500, **Plan B** gives the higher cumulative deposit after 10 months.

Q20. AP: first term a, $d = 6$. GP: first term a, $r = 2$.

(i) **n-th term expressions:**

$$T_n(\text{AP}) = a + (n-1)(6) = a + 6n - 6$$
$$T_n(\text{GP}) = a \cdot 2^{n-1}$$

(ii) **Setting equal and deriving the equation:**

$$a + 6(n-1) = a \cdot 2^{n-1}$$
$$a \cdot 2^{n-1} - 6n + 6 - a = 0$$
$$a \cdot 2^{n-1} - 6n = a - 6 \qquad \checkmark$$

(iii) **Verify $n = 1$ satisfies the equation for all a:**

Substituting $n = 1$:

$$\text{LHS} = a \cdot 2^0 - 6(1) = a - 6$$
$$\text{RHS} = a - 6$$

LHS = RHS for **all** values of a. ✓

(iv) **With $a = 6$, test $n = 1, 2, 3, 4$:**

The equation becomes $6 \cdot 2^{n-1} = 6 + 6(n-1) = 6n$, i.e., $2^{n-1} = n$.

$$n = 1: \quad 2^0 = 1 = 1 \quad \checkmark$$
$$n = 2: \quad 2^1 = 2 = 2 \quad \checkmark$$
$$n = 3: \quad 2^2 = 4 \neq 3 \quad \times$$
$$n = 4: \quad 2^3 = 8 \neq 4 \quad \times$$

Conclusion: For $a = 6$, the n-th terms are equal at $\mathbf{n = 1}$ and $\mathbf{n = 2}$ only.

Q21. AP: first term a, common difference d ($d \neq 0$).

(i) **Relevant AP terms:**

$$T_2 = a + d$$
$$T_5 = a + 4d$$
$$T_{14} = a + 13d$$

(ii) **GP condition — square of middle term equals product of the other two:**

$$(T_5)^2 = T_2 \cdot T_{14}$$
$$(a + 4d)^2 = (a + d)(a + 13d)$$

Expanding:

$$\text{LHS:} \quad a^2 + 8ad + 16d^2$$
$$\text{RHS:} \quad a^2 + 13ad + ad + 13d^2 = a^2 + 14ad + 13d^2$$

Setting LHS = RHS:

$$a^2 + 8ad + 16d^2 = a^2 + 14ad + 13d^2$$
$$16d^2 - 13d^2 = 14ad - 8ad$$
$$3d^2 = 6ad$$

(iii) **Relationship between a and d; common ratio r:**
Since $d \neq 0$, dividing both sides by $3d$:

$$d = 2a \qquad \Longrightarrow \qquad a = \frac{d}{2}$$

Common ratio of the GP:

$$r = \frac{T_5}{T_2} = \frac{a + 4d}{a + d}$$

Substituting $a = \dfrac{d}{2}$:

$$r = \frac{\frac{d}{2} + 4d}{\frac{d}{2} + d} = \frac{\frac{9d}{2}}{\frac{3d}{2}} = \frac{9d}{3d} = \mathbf{3}$$

(iv) **Finding a and d from $S_5 = 80$:**

$$S_5 = \frac{5}{2}\big[2a + 4d\big] = 80$$
$$2a + 4d = 32$$
$$a + 2d = 16$$

Substituting $d = 2a$:

$$\begin{aligned} a + 2(2a) &= 16 \\ 5a &= 16 \\ a &= \frac{16}{5}, \qquad d = 2a = \frac{32}{5} \end{aligned}$$

First three terms of the GP:

$$\begin{aligned} T_2 &= a + d = \frac{16}{5} + \frac{32}{5} = \frac{48}{5} \\ T_5 &= a + 4d = \frac{16}{5} + \frac{128}{5} = \frac{144}{5} \\ T_{14} &= a + 13d = \frac{16}{5} + \frac{416}{5} = \frac{432}{5} \end{aligned}$$

Verification of GP: $\dfrac{144/5}{48/5} = 3$ and $\dfrac{432/5}{144/5} = 3$ ✓

Sum of the first 5 terms of the GP (first term $= \dfrac{48}{5}$, $r = 3$):

$$S_5(\text{GP}) = \frac{48}{5} \times \frac{3^5 - 1}{3 - 1} = \frac{48}{5} \times \frac{242}{2} = \frac{48}{5} \times 121 = \frac{5808}{5} = \mathbf{1161.6}$$