Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (C) 65°

Reason: The angle subtended by an arc at the centre is twice the angle subtended at any point on the remaining part of the circle.

$$\angle BOC = 130^\circ$$
$$\angle BAC = \frac{1}{2}\angle BOC = \frac{130^\circ}{2} = \mathbf{65}^\circ$$

Q2. Correct Option: (C) 55°

Reason: Since AB is a diameter, $\angle ACB = 90^\circ$ (angle in a semicircle). In $\triangle ABC$:

$$\begin{aligned}\angle ABC &= 180^\circ - \angle BAC - \angle ACB \\ &= 180^\circ - 35^\circ - 90^\circ = \mathbf{55}^\circ\end{aligned}$$

Q3. Correct Option: (A) 85° **and** 102°

Reason: In a cyclic quadrilateral, opposite angles are supplementary.

$$\angle P + \angle R = 180^\circ \implies \angle R = 180^\circ - 78^\circ = \mathbf{102}^\circ$$
$$\angle Q + \angle S = 180^\circ \implies \angle S = 180^\circ - 95^\circ = \mathbf{85}^\circ$$

Q4. Correct Option: (C) 12 **cm**

Reason: The radius $OT \perp PT$ (tangent $\perp$ radius at point of contact). In right $\triangle OTP$:

$$PT = \sqrt{OP^2 - OT^2} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = \mathbf{12} \text{ cm}$$

Q5. Correct Option: (C) 120°

Reason: Since $OA \perp PA$ and $OB \perp PB$ (radius $\perp$ tangent):

$$\angle OAP = \angle OBP = 90^\circ$$

In quadrilateral $OAPB$, angle sum $= 360^\circ$:

$$\angle AOB = 360^\circ - 90^\circ - 90^\circ - 60^\circ = \mathbf{120}^\circ$$

Q6. Correct Option: (C) 6

Reason: When two chords intersect inside a circle, the products of their segments are equal:

$$PA \times PB = PC \times PD$$
$$3 \times 8 = 4 \times PD$$
$$PD = \frac{24}{4} = \mathbf{6}$$

Q7. Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Reason:

Assertion A is TRUE. $\angle TQR$ (angle between tangent TQ and chord QR) equals the angle subtended by chord QR in the alternate segment — this is a standard result.

Reason R is TRUE. R states precisely the **Alternate Segment Theorem** (Tangent-Chord angle = angle in alternate segment), which is exactly the theorem that proves A. Hence R is the correct explanation of A.

Q8. Correct Option: (A) 50°

Reason: $\angle AOC = 100^\circ$ is the central angle subtended by arc AC (minor arc).

Point D lies on the **major arc**. The angle subtended at the centre by the minor arc $AC = 100^\circ$, so the angle subtended by the major arc AC at the centre $= 360^\circ - 100^\circ = 260^\circ$.

By the inscribed angle theorem, the angle subtended at D on the major arc equals **half the central angle of the minor arc**:

$$\angle ADC = \frac{1}{2} \times \angle AOC = \frac{100^\circ}{2} = \mathbf{50}^\circ$$

Q8. Correct Option: (A) 50°

Reason: By the inscribed angle theorem, the angle at D (on the major arc) $= \frac{1}{2}\angle AOC = \frac{100^\circ}{2} = \mathbf{50}^\circ$.

Q9. Correct Option: (B) 75°

Reason: In cyclic quadrilateral $ABCD$, opposite angles are supplementary:

$$\angle BCD = 180^\circ - \angle DAB = 180^\circ - 75^\circ = 105^\circ$$

Since DCE is a straight line:

$$\angle BCE = 180^\circ - \angle BCD = 180^\circ - 105^\circ = \mathbf{75}^\circ$$

Q10. Correct Option: (C) 16 **cm**

Reason: By the tangent-secant theorem (from external point P):

$$PT^2 = PA \times PB$$

$$8^2 = 4 \times PB$$

$$PB = \frac{64}{4} = \mathbf{16} \text{ cm}$$

Q11. Correct Option: (A) 8 **cm and** 0 **cm**

Reason:

Distance between centres: When two circles touch externally, the distance between their centres = sum of radii:

$$d = 5 + 3 = \mathbf{8} \text{ cm}$$

Length of common tangent at point of contact: The common tangent at the point of contact is the internal common tangent. Since the two circles meet at exactly one point, the length of this tangent between the two circles is **0** cm (it is a single point, not a segment).

Section B — Short Answer Questions

Q12. Given: $\angle AOB = 112°$; O is the centre.

(i) $\angle ACB$**:**
By the inscribed angle theorem, the angle at the centre is twice the angle at the circumference subtended by the same arc AB:

$$\angle ACB = \frac{1}{2}\angle AOB = \frac{112°}{2} = \mathbf{56}°$$

(ii) $\angle ADB$**:**
D also lies on the remaining arc (same side as C). Both C and D subtend arc AB from the same side, so:

$$\angle ADB = \angle ACB = \mathbf{56}°$$

(iii) **Theorem connecting (i) and (ii):**
Angles in the same segment theorem: Angles subtended by the same arc (or chord) at any two points on the same side of the chord on the circle are equal. Hence $\angle ACB = \angle ADB$.

Q13. Cyclic quadrilateral $ABCD$: $\angle DAB = 3x + 5°$, $\angle BCD = 2x + 15°$, $\angle ABC = 85°$.

(i) **Value of x:**
Opposite angles of a cyclic quadrilateral are supplementary:

$$\begin{aligned} \angle DAB + \angle BCD &= 180° \\ (3x + 5) + (2x + 15) &= 180 \\ 5x + 20 &= 180 \\ 5x &= 160 \\ x &= \mathbf{32} \end{aligned}$$

(ii) $\angle DAB$ **and** $\angle BCD$**:**

$$\angle DAB = 3(32) + 5 = \mathbf{101}°$$

$$\angle BCD = 2(32) + 15 = \mathbf{79}°$$

Verification: $101° + 79° = 180°$ ✓
Also: $\angle ABC + \angle ADC = 180°$:

$$\angle ADC = 180° - 85° = 95°$$

(iii) **Exterior angle $\angle ADE$ (CD produced to E):**
The exterior angle of a cyclic quadrilateral equals the interior opposite angle:

$$\angle ADE = \angle ABC = \mathbf{85}^\circ$$

Alternatively: $\angle ADE = 180^\circ - \angle ADC = 180^\circ - 95^\circ = 85^\circ$ ✓

Given: Radius $OA = OB = 6$ cm; $OP = 10$ cm; PA and PB are tangents.

Q14. (i) **Length of tangents PA and PB:**
Since radius $\perp$ tangent at point of contact ($OA \perp PA$):

$$PA = \sqrt{OP^2 - OA^2} = \sqrt{100 - 36} = \sqrt{64} = \mathbf{8} \text{ cm}$$

Since tangents from an external point are equal: $PB = PA = \mathbf{8}$ cm.

(ii) $\angle OAP$**:**
In right $\triangle OAP$ ($\angle OAP = 90^\circ$):

$$\sin(\angle OPA) = \frac{OA}{OP} = \frac{6}{10} = 0.6 \implies \angle OPA = 36.87^\circ$$
$$\angle OAP = \mathbf{90}^\circ$$

(iii) $\angle AOB$ **given** $\angle APB = 73^\circ$**:**
In quadrilateral $OAPB$:

$$\angle OAP + \angle OBP = 90^\circ + 90^\circ = 180^\circ$$
$$\angle AOB + \angle APB = 360^\circ - 180^\circ = 180^\circ$$
$$\angle AOB = 180^\circ - 73^\circ = \mathbf{107}^\circ$$

Q15. Chords AB and CD intersect at P inside the circle. $AP = 4$ cm, $PB = 9$ cm, $CP = 6$ cm.

(i) **Intersecting Chords Theorem:**
When two chords intersect inside a circle, the products of their segments are equal:

$$AP \times PB = CP \times PD$$

(ii) **Finding PD:**

$$4 \times 9 = 6 \times PD$$
$$PD = \frac{36}{6} = \mathbf{6} \text{ cm}$$

(iii) **Full lengths of AB and CD:**

$$AB = AP + PB = 4 + 9 = \mathbf{13} \text{ cm}$$
$$CD = CP + PD = 6 + 6 = \mathbf{12} \text{ cm}$$

TQ is tangent at Q; chord QR; $\angle TQR = 48°$; S on major arc.

Q16. (i) $\angle QSR$**:**

By the **Alternate Segment Theorem**, the angle between tangent TQ and chord QR equals the angle subtended by chord QR in the alternate segment:

$$\angle QSR = \angle TQR = \mathbf{48}°$$

(ii) $\angle QRS$**, given** $\angle SQR = 62°$**:**

In $\triangle QSR$, angle sum $= 180°$:

$$\begin{aligned}\angle QRS &= 180° - \angle QSR - \angle SQR \\ &= 180° - 48° - 62° = \mathbf{70}°\end{aligned}$$

(iii) **Angle between tangent** TQ **and chord** QS**, given** $\angle QRS = 70°$**:**

By the **Alternate Segment Theorem**, the angle between tangent TQ and chord QS equals the angle in the alternate segment, which is $\angle QRS$:

$$\angle(\text{between } TQ \text{ and } QS) = \angle QRS = \mathbf{70}°$$

Section C — Long Answer Questions

Q17. AB is diameter; $\angle CAB = 38°$; $\angle ACD = 27°$; tangent TP at B.

(i) $\angle ADB$**:**

Since AB is a diameter, $\angle ACB = 90°$ (angle in semicircle).

$\angle ADB$ subtends arc AB (diameter) from point D:

$$\angle ADB = 90°$$

Theorem: Angle in a semicircle is 90°.

(ii) $\angle ABD$**:**

In $\triangle ABD$, $\angle ADB = 90°$:

$$\angle ABD = 90° - \angle DAB$$

$\angle DAB = \angle CAB + \angle CAD = 38° + 27° = 65°$ (from the figure, D is between A and C on the arc, so $\angle DAB = \angle CAB -$ ...).

Re-examining: $\angle CAB = 38°$ subtends arc BC. By alternate angles in the same segment, $\angle ADB = 90°$ (diameter). In $\triangle ABD$:

$$\angle ABD = 180° - 90° - \angle DAB$$

$\angle DAB$ subtends arc DB. Using $\angle ACD = 27°$ (subtends arc AD), and angles in the same segment: $\angle ABD = \angle ACD = \mathbf{27}°$ (both subtend arc AD).

(iii) $\angle ADC$ **(opposite angle in cyclic quadrilateral):**

$$\begin{aligned}\angle ADC + \angle ABC &= 180° \\ \angle ABC &= \angle ABD + \angle DBC\end{aligned}$$

$\angle DBC = \angle DAC$ (same segment, subtend arc DC) $= 27°$ ... Using $\angle ACB = 90°$ and $\angle CAB = 38°$:

$$\angle ABC = 90° - 38° = 52°$$

$$\angle ADC = 180° - \angle ABC = 180° - 52° = \mathbf{128}°$$

(iv) $\angle TBC$ **by Alternate Segment Theorem:**
By the Alternate Segment Theorem, the angle between tangent TB and chord BC equals the angle in the alternate segment $\angle BDC$ (or $\angle BAC$ for chord BC):

$$\angle TBC = \angle CAB = \mathbf{38}°$$

Q18. PA tangent at A; BC diameter; AC chord; $\angle PAC = 50°$.

(i) $\angle ABC$ **by Alternate Segment Theorem:**
The angle between tangent PA and chord AC equals the angle subtended by chord AC in the alternate segment:

$$\angle ABC = \angle PAC = \mathbf{50}°$$

(ii) $\angle BAC$ **(angle in semicircle):**
Since BC is a diameter:

$$\angle BAC = \mathbf{90}°$$

Theorem: Angle in a semicircle is $90°$.

(iii) $\angle ACB$**:**
In $\triangle ABC$, angle sum $= 180°$:

$$\angle ACB = 180° - \angle BAC - \angle ABC$$
$$= 180° - 90° - 50° = \mathbf{40}°$$

(iv) $\angle AOC$ **by angle-at-centre property:**
The angle subtended by arc AC at the centre is twice the angle subtended at the circumference (at B):

$$\angle AOC = 2 \times \angle ABC = 2 \times 50° = \mathbf{100}°$$