Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (B) $m = \frac{3}{2},\ c = 4$

Reason: Rearranging $3x - 2y + 8 = 0$ into slope-intercept form:

$$\begin{aligned} 3x - 2y + 8 &= 0 \\ 2y &= 3x + 8 \\ y &= \frac{3}{2}x + 4 \end{aligned}$$

Hence $m = \frac{3}{2}$ and $c = 4$.

Q2. Correct Option: (B) -1

Reason: The slope of a line making angle θ with the positive x-axis is $m = \tan\theta$.

$$m = \tan 135^\circ = \tan(180^\circ - 45^\circ) = -\tan 45^\circ = -1$$

Q3. Correct Option: (C) $(0,\ -7)$

Reason: A line crosses the y-axis where $x = 0$. Substituting $x = 0$ into $y = 5x - 7$:

$$y = 5(0) - 7 = -7$$

The line crosses the y-axis at $(0,\ -7)$.

Q4. Correct Option: (A) $y = 2x - 7$

Reason: Using the two points $(2,\ -3)$ and $(4,\ 1)$:

Step 1 — Find the slope:

$$m = \frac{1 - (-3)}{4 - 2} = \frac{4}{2} = 2$$

Step 2 — Use point-slope form with $(2,\ -3)$:

$$\begin{aligned} y - (-3) &= 2(x - 2) \\ y + 3 &= 2x - 4 \\ y &= 2x - 7 \end{aligned}$$

Q5. Correct Option: (B) $4x - 6y + 1 = 0$

Reason: Parallel lines have equal slopes. Rewriting the given line $2x - 3y + 5 = 0$:

$$y = \frac{2}{3}x + \frac{5}{3} \implies m = \frac{2}{3}$$

Checking option (B): $4x - 6y + 1 = 0 \implies y = \frac{4}{6}x + \frac{1}{6} = \frac{2}{3}x + \frac{1}{6}$.

Slope $= \frac{2}{3}$ (same as given line) and y-intercept $\frac{1}{6} \neq \frac{5}{3}$ (different), so the lines are parallel. ✓

Q6. Correct Option: (D) $-\dfrac{1}{3}$

Reason: For two perpendicular lines with slopes m_1 and m_2, the condition is $m_1 \cdot m_2 = -1$.

The given line $y = 3x + 4$ has slope $m_1 = 3$.

$$m_2 = \frac{-1}{m_1} = \frac{-1}{3}$$

Q7. Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Reason:

Checking Assertion A:

Rewrite both lines in slope-intercept form:

$$4x + 6y = 9 \implies y = -\frac{2}{3}x + \frac{3}{2}, \quad m_1 = -\frac{2}{3}, \quad c_1 = \frac{3}{2}$$
$$6x + 9y = 15 \implies y = -\frac{2}{3}x + \frac{5}{3}, \quad m_2 = -\frac{2}{3}, \quad c_2 = \frac{5}{3}$$

Since $m_1 = m_2 = -\dfrac{2}{3}$ and $c_1 \neq c_2$, the lines are parallel. **Assertion A is TRUE.**

Reason R correctly states that parallel lines have equal slopes and different y-intercepts, which is precisely the condition verified above. **R is TRUE and is the correct explanation of A.**

Q8. Correct Option: (B) ℓ_1 has slope 1 and ℓ_2 has slope -1; the lines are perpendicular to each other.

Reason: From the graph:

- ℓ_1 passes through $(-2, 0)$ and $(0, 2)$:

$$m_1 = \frac{2 - 0}{0 - (-2)} = \frac{2}{2} = 1$$

- ℓ_2 passes through $(0, 2)$ and $(2, 0)$:

$$m_2 = \frac{0 - 2}{2 - 0} = \frac{-2}{2} = -1$$

Both lines share the same y-intercept $(0, 2)$ but have slopes 1 and -1. Since $m_1 \times m_2 = 1 \times (-1) = -1$, the lines are **perpendicular**.

Q9. Correct Option: (A) $x - 2y - 7 = 0$

Reason: The given line is $x - 2y + 7 = 0$, with slope $m = \dfrac{1}{2}$. A parallel line has the same slope.

Using point-slope form with $(3, -2)$ and $m = \frac{1}{2}$:

$$\begin{aligned} y - (-2) &= \frac{1}{2}(x - 3) \\ y + 2 &= \frac{1}{2}x - \frac{3}{2} \\ 2y + 4 &= x - 3 \\ x - 2y - 7 &= 0 \quad \checkmark \end{aligned}$$

Q10. **Correct Option: (C)** 1 **or** -1

Reason: A line equally inclined to both axes makes an angle of $45°$ or $135°$ with the positive x-axis.

$$\tan 45° = 1$$
$$\tan 135° = -1$$

Hence the possible slopes are 1 or -1.

Q11. **Correct Option: (A)** $y = x$

Reason:

Find slope of AB**:**

$$m_{AB} = \frac{-1-3}{5-1} = \frac{-4}{4} = -1$$

Find slope of the required line (perpendicular to AB**):**

$$m = \frac{-1}{m_{AB}} = \frac{-1}{-1} = 1$$

Equation through the origin with slope 1**:**

$$y - 0 = 1(x - 0)$$
$$y = x$$

Section B — Short Answer Questions

Q12. Given: slope $m = -\frac{2}{3}$, passes through $(0, 5)$.

(i) **Equation in the form** $y = mx + c$**:**
Since the line passes through $(0, 5)$, the y-intercept is $c = 5$.

$$\boxed{y = -\frac{2}{3}x + 5}$$

(ii) **x-intercept (set $y = 0$):**

$$\begin{aligned} 0 &= -\frac{2}{3}x + 5 \\ \frac{2}{3}x &= 5 \\ x &= \frac{15}{2} = 7.5 \end{aligned}$$

The x-intercept is $\left(\frac{15}{2},\ 0\right)$.

(iii) **Angle θ with the positive x-axis:**
Since $m = -\frac{2}{3} < 0$, the line makes an obtuse angle with the positive x-axis.

$$\begin{aligned} |m| &= \frac{2}{3} \\ \tan^{-1}\left(\frac{2}{3}\right) &\approx 33.7^\circ \end{aligned}$$

Since the slope is negative:

$$\theta = 180^\circ - 33.7^\circ = \mathbf{146.3^\circ} \approx \mathbf{146^\circ}$$

Q13. Given: $A(-1,\ 4)$ and $B(3,\ -4)$.

(i) **Equation in the form $y = mx + c$:**
Step 1 — Find the slope:

$$m = \frac{-4 - 4}{3 - (-1)} = \frac{-8}{4} = -2$$

Step 2 — Use point-slope form with $A(-1,\ 4)$:

$$\begin{aligned} y - 4 &= -2(x - (-1)) \\ y - 4 &= -2x - 2 \\ y &= -2x + 2 \end{aligned}$$

$$\boxed{y = -2x + 2}$$

Verification with $B(3,\ -4)$: $y = -2(3) + 2 = -4$ ✓

(ii) **Slope and y-intercept:**

$$\text{Slope } m = -2, \qquad y\text{-intercept } c = 2$$

(iii) **x-intercept (set $y = 0$):**

$$\begin{aligned} 0 &= -2x + 2 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

The line crosses the x-axis at $(\mathbf{1},\ \mathbf{0})$.

Q14. Given: $3x - ky + 6 = 0$ is perpendicular to $4x + 3y - 1 = 0$.

(i) **Finding k:**
Slope of $3x - ky + 6 = 0$:

$$ky = 3x + 6 \implies m_1 = \frac{3}{k}$$

Slope of $4x + 3y - 1 = 0$:

$$3y = -4x + 1 \implies m_2 = -\frac{4}{3}$$

Condition for perpendicularity: $m_1 \times m_2 = -1$:

$$\frac{3}{k} \times \left(-\frac{4}{3}\right) = -1$$
$$\frac{-4}{k} = -1$$
$$k = 4$$

$$\boxed{k = 4}$$

(ii) **Equation of the first line with $k = 4$:**

$$3x - 4y + 6 = 0$$
$$4y = 3x + 6$$
$$y = \frac{3}{4}x + \frac{3}{2}$$

(iii) **y-intercept of the first line:**
Setting $x = 0$:

$$y = \frac{3}{4}(0) + \frac{3}{2} = \frac{3}{2}$$

The y-intercept is $\left(0,\ \frac{3}{2}\right)$.

Q15. Given: $\ell_1 : (k+1)x - 3y = 7$ and $\ell_2 : 2x - 6y + 5 = 0$ are parallel.

(i) **Finding k:**
Slope of ℓ_1:

$$3y = (k+1)x - 7 \implies m_1 = \frac{k+1}{3}$$

Slope of ℓ_2:

$$6y = 2x + 5 \implies m_2 = \frac{2}{6} = \frac{1}{3}$$

Condition for parallel lines $m_1 = m_2$:

$$\frac{k+1}{3} = \frac{1}{3}$$
$$k+1 = 1$$
$$k = 0$$

$$\boxed{k = 0}$$

(ii) **Equation of ℓ_1 in slope-intercept form:**
Substituting $k = 0$ into $(k+1)x - 3y = 7$:

$$x - 3y = 7$$
$$3y = x - 7$$
$$y = \frac{1}{3}x - \frac{7}{3}$$

(iii) **Verification that ℓ_1 and ℓ_2 do not coincide:**

$$\ell_1: \quad y = \frac{1}{3}x - \frac{7}{3}, \quad c_1 = -\frac{7}{3}$$
$$\ell_2: \quad y = \frac{1}{3}x + \frac{5}{6}, \quad c_2 = \frac{5}{6}$$

Since $c_1 = -\frac{7}{3} \neq \frac{5}{6} = c_2$, the lines have different y-intercepts. Hence ℓ_1 and ℓ_2 are parallel but **do not coincide**. ✓

Q16. Given: Line ℓ cuts the x-axis at $(-3, 0)$ and the y-axis at $(0, 4)$.

(i) **Slope of the line:**

$$m = \frac{4-0}{0-(-3)} = \frac{4}{3}$$

(ii) **Equation in slope-intercept form:**
The y-intercept is $c = 4$ (since the line passes through $(0, 4)$).

$$\boxed{y = \frac{4}{3}x + 4}$$

Verification with $(-3, 0)$*:* $y = \frac{4}{3}(-3) + 4 = -4 + 4 = 0$ ✓

(iii) **Length of the segment between the coordinate axes:**
The segment joins $(-3, 0)$ and $(0, 4)$.

$$\text{Length} = \sqrt{(0-(-3))^2 + (4-0)^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25} = \mathbf{5} \text{ units}$$

Section C — Long Answer Questions

Q17. Given: $C_1(2,\,5)$ and $C_2(8,\,9)$.

(i) **Slope and equation of the road:**

$$m = \frac{9-5}{8-2} = \frac{4}{6} = \frac{2}{3}$$

Using point-slope form with $C_1(2,\,5)$:

$$y - 5 = \frac{2}{3}(x-2)$$
$$y = \frac{2}{3}x - \frac{4}{3} + 5 = \frac{2}{3}x + \frac{11}{3}$$
$$\boxed{y = \frac{2}{3}x + \frac{11}{3}}$$

(ii) **Point where road crosses the y-axis (set $x = 0$):**

$$y = \frac{2}{3}(0) + \frac{11}{3} = \frac{11}{3}$$

The petrol pump is at $\left(0,\,\dfrac{11}{3}\right)$.

Interpretation: This point is approximately 3.67 km along the y-axis from the origin, representing the location on the map where the road crosses the north–south reference line.

(iii) **Equation of perpendicular lane at $C_1(2,\,5)$:**

The perpendicular slope is $m_\perp = -\dfrac{1}{m} = -\dfrac{3}{2}$.

$$y - 5 = -\frac{3}{2}(x-2)$$
$$y = -\frac{3}{2}x + 3 + 5 = -\frac{3}{2}x + 8$$
$$\boxed{y = -\frac{3}{2}x + 8}$$

(iv) **Comparing with $3x - 2y = 4$:**

$$3x - 2y = 4 \implies y = \frac{3}{2}x - 2, \quad m_2 = \frac{3}{2}$$

Main road slope: $m_1 = \dfrac{2}{3}$.

$$m_1 \neq m_2 \qquad \implies \text{not parallel}$$
$$m_1 \times m_2 = \frac{2}{3} \times \frac{3}{2} = 1 \neq -1 \qquad \implies \text{not perpendicular}$$

The two roads are **neither parallel nor perpendicular** to each other.

Q18. Given: AB lies on $y = 2x + 1$; $C(7,\, 2)$; $D(5,\, -2)$.

(i) **Slope of AB:**
Since AB lies on $y = 2x + 1$:

$$\text{slope of } AB = m_{AB} = 2$$

(ii) **Equation of line BC (perpendicular to AB through $C(7,\, 2)$):**

$$m_{BC} = -\frac{1}{m_{AB}} = -\frac{1}{2}$$

$$y - 2 = -\frac{1}{2}(x - 7)$$
$$y = -\frac{1}{2}x + \frac{7}{2} + 2 = -\frac{1}{2}x + \frac{11}{2}$$
$$\boxed{y = -\frac{1}{2}x + \frac{11}{2}}$$

(iii) **Equation of line AD (parallel to BC through $D(5,\, -2)$):**
Since $AD \parallel BC$, slope $= -\frac{1}{2}$.

$$y - (-2) = -\frac{1}{2}(x - 5)$$
$$y + 2 = -\frac{1}{2}x + \frac{5}{2}$$
$$y = -\frac{1}{2}x + \frac{1}{2}$$
$$\boxed{y = -\frac{1}{2}x + \frac{1}{2}}$$

(iv) **Co-ordinates of vertex B (intersection of AB and BC):**
Setting $y = 2x + 1$ equal to $y = -\frac{1}{2}x + \frac{11}{2}$:

$$2x + 1 = -\frac{1}{2}x + \frac{11}{2}$$
$$2x + \frac{1}{2}x = \frac{11}{2} - 1 = \frac{9}{2}$$
$$\frac{5}{2}x = \frac{9}{2}$$
$$x = \frac{9}{5}$$

$$y = 2\left(\frac{9}{5}\right) + 1 = \frac{18}{5} + \frac{5}{5} = \frac{23}{5}$$

$$B = \left(\frac{9}{5},\, \frac{23}{5}\right)$$

Q19. (i) **Equations of ℓ_A, ℓ_B, ℓ_C:**
Aarav's line ℓ_A: Through origin, slope $= \tan 45° = 1$:

$$\ell_A : \quad y = x$$

Bina's line ℓ_B: Through (0, 4), parallel to x-axis (slope $= 0$):

$$\ell_B : \quad y = 4$$

Chetan's line ℓ_C: Through (6, 0), perpendicular to ℓ_A (slope $= -1$):

$$y - 0 = -1(x - 6) \implies y = -x + 6$$

$$\ell_C : \quad y = -x + 6$$

(ii) **Intersection of ℓ_A and ℓ_C:**

$$x = -x + 6 \implies 2x = 6 \implies x = 3, \quad y = 3$$

$$\ell_A \cap \ell_C = (\mathbf{3},\ \mathbf{3})$$

(iii) **Intersection of ℓ_B and ℓ_C:**

$$4 = -x + 6 \implies x = 2, \quad y = 4$$

$$\ell_B \cap \ell_C = (\mathbf{2},\ \mathbf{4})$$

(iv) **Vertices of the triangle and perpendicular sides:**
The three vertices are the pairwise intersections of the three lines:

$$\begin{aligned} \ell_A \cap \ell_B &: \quad y = x \text{ and } y = 4 \implies x = 4 \implies (4,\ 4) \\ \ell_A \cap \ell_C &: \quad (3,\ 3) \\ \ell_B \cap \ell_C &: \quad (2,\ 4) \end{aligned}$$

Vertices: (4, 4), (3, 3), (2, 4).
Since $m_{\ell_A} \times m_{\ell_C} = 1 \times (-1) = -1$, the sides lying on ℓ_A and ℓ_C are **perpendicular** to each other.

Q20. Given: $\ell_1 : 4x - 3y + 12 = 0$.

(i) **Slope-intercept form, slope, y-intercept, x-intercept of ℓ_1:**

$$3y = 4x + 12 \implies y = \frac{4}{3}x + 4$$

$$\text{Slope: } m_1 = \frac{4}{3}, \qquad y\text{-intercept: } c = 4 \implies (0,\ 4)$$

x-intercept (set $y = 0$):

$$0 = \frac{4}{3}x + 4 \implies x = -3 \implies (-3,\ 0)$$

(ii) **Equation of ℓ_2 (perpendicular to ℓ_1, through $(-3, 0)$):**

$$m_2 = -\frac{1}{m_1} = -\frac{3}{4}$$

$$y - 0 = -\frac{3}{4}(x + 3)$$
$$y = -\frac{3}{4}x - \frac{9}{4}$$
$$\boxed{\ell_2 : \quad y = -\frac{3}{4}x - \frac{9}{4}}$$

(iii) **Equation of ℓ_3 (parallel to ℓ_1, through $(2, 3)$):**

$$m_3 = m_1 = \frac{4}{3}$$

$$y - 3 = \frac{4}{3}(x - 2)$$
$$y = \frac{4}{3}x - \frac{8}{3} + 3 = \frac{4}{3}x + \frac{1}{3}$$
$$\boxed{\ell_3 : \quad y = \frac{4}{3}x + \frac{1}{3}}$$

y-intercept of $\ell_3 = \frac{1}{3}$, i.e., the point $\left(0, \frac{1}{3}\right)$.

(iv) **Verification that $\ell_2 \perp \ell_3$:**

$$m_2 \times m_3 = \left(-\frac{3}{4}\right) \times \frac{4}{3} = -\frac{12}{12} = -1 \quad \checkmark$$

Since the product of slopes equals -1, ℓ_2 is indeed perpendicular to ℓ_3.

Q21. Given: $P(1, 2)$, $Q(5, 6)$, $R(7, -2)$.

(i) **Perpendicular from P to side QR:**
Slope of QR:

$$m_{QR} = \frac{-2 - 6}{7 - 5} = \frac{-8}{2} = -4$$

Slope of perpendicular from P:

$$m = \frac{-1}{-4} = \frac{1}{4}$$

Equation through $P(1, 2)$:

$$y - 2 = \frac{1}{4}(x - 1)$$
$$4y - 8 = x - 1$$
$$x - 4y + 7 = 0$$
$$\boxed{x - 4y + 7 = 0}$$

(ii) **Perpendicular from Q to side PR:**
Slope of PR:

$$m_{PR} = \frac{-2-2}{7-1} = \frac{-4}{6} = -\frac{2}{3}$$

Slope of perpendicular from Q:

$$m = \frac{-1}{-\frac{2}{3}} = \frac{3}{2}$$

Equation through $Q(5,\, 6)$:

$$\begin{aligned} y - 6 &= \frac{3}{2}(x-5) \\ 2y - 12 &= 3x - 15 \\ 3x - 2y - 3 &= 0 \end{aligned}$$

$$\boxed{3x - 2y - 3 = 0}$$

(iii) **Orthocentre H (intersection of the two perpendiculars):**
From (i): $x = 4y - 7$. Substituting into (ii):

$$\begin{aligned} 3(4y-7) - 2y - 3 &= 0 \\ 12y - 21 - 2y - 3 &= 0 \\ 10y &= 24 \\ y &= \frac{12}{5} \end{aligned}$$

$$x = 4\left(\frac{12}{5}\right) - 7 = \frac{48}{5} - \frac{35}{5} = \frac{13}{5}$$

$$\boxed{H = \left(\frac{13}{5},\, \frac{12}{5}\right)}$$

(iv) **Verification via perpendicular from R to PQ:**
Slope of PQ:

$$m_{PQ} = \frac{6-2}{5-1} = \frac{4}{4} = 1$$

Slope of perpendicular from R:

$$m = -1$$

Equation through $R(7,\, -2)$:

$$\begin{aligned} y - (-2) &= -1(x-7) \\ y + 2 &= -x + 7 \\ x + y - 5 &= 0 \end{aligned}$$

Check whether $H\left(\frac{13}{5}, \frac{12}{5}\right)$ satisfies $x + y - 5 = 0$:

$$\frac{13}{5} + \frac{12}{5} - 5 = \frac{25}{5} - 5 = 5 - 5 = 0 \quad \checkmark$$

The perpendicular from R to PQ passes through H, confirming that $H = \left(\frac{13}{5}, \frac{12}{5}\right)$ is the orthocentre of triangle PQR.