Solutions: Section A — Multiple Choice Questions

Q1. Remainder Theorem Concept

Answer: (B) $f(a)$
Solution: By definition of the Remainder Theorem, when a polynomial $f(x)$ is divided by a linear divisor of the form $(x - a)$, the remainder is the value of the polynomial at $x = a$. Thus, the remainder is $f(a)$.

Q2. Finding the value of k

Answer: (B) $k = 3$
Solution: Given $f(x) = x^3 - 3x^2 + kx - 2$.
According to the Factor Theorem, if $(x - 2)$ is a factor, then $f(2) = 0$.

$$\begin{aligned}(2)^3 - 3(2)^2 + k(2) - 2 &= 0\\ 8 - 3(4) + 2k - 2 &= 0\\ 8 - 12 + 2k - 2 &= 0\\ 2k - 6 &= 0\\ 2k &= 6 \implies k = 3\end{aligned}$$

Q3. Calculation of Remainder

Answer: (A) -11
Solution: Let $f(x) = 2x^3 - 5x^2 + 3x - 1$.
To find the remainder when divided by $(x + 1)$, we evaluate $f(-1)$:

$$\begin{aligned}f(-1) &= 2(-1)^3 - 5(-1)^2 + 3(-1) - 1\\ &= 2(-1) - 5(1) - 3 - 1\\ &= -2 - 5 - 3 - 1 = -11\end{aligned}$$

Q4. Factor Identification

Answer: (C) $(x - 2)$
Solution: We check the options using the Factor Theorem. For option (C), we evaluate $f(2)$:

$$\begin{aligned}f(2) &= (2)^3 + 2(2)^2 - 5(2) - 6\\ &= 8 + 8 - 10 - 6\\ &= 16 - 16 = 0\end{aligned}$$

Since $f(2) = 0$, $(x - 2)$ is a factor.

Q5. Factorisation of a Cubic Polynomial

Answer: (A) $(x - 1)(x - 2)(x - 3)$
Solution: Let $f(x) = x^3 - 6x^2 + 11x - 6$. By inspection:

- $f(1) = 1 - 6 + 11 - 6 = 0 \implies (x - 1)$ is a factor.
- $f(2) = 8 - 24 + 22 - 6 = 0 \implies (x - 2)$ is a factor.

- $f(3) = 27 - 54 + 33 - 6 = 0 \implies (x-3)$ is a factor.

Thus, the factors are $(x-1)(x-2)(x-3)$.

Q6. Solving for unknown coefficient a

Answer: (D) $\frac{4}{3}$
Solution: If $(x+3)$ is a factor of $g(x) = x^3 + ax^2 - 4x + 3$, then $g(-3) = 0$.

$$\begin{aligned} (-3)^3 + a(-3)^2 - 4(-3) + 3 &= 0 \\ -27 + 9a + 12 + 3 &= 0 \\ 9a - 12 &= 0 \\ 9a = 12 \implies a = \frac{12}{9} &= \frac{4}{3} \end{aligned}$$

Q7. Assertion and Reason

Answer: (A)
Solution: Reason (R): The Factor Theorem states $(x-1)$ is a factor if $f(1) = 0$. This is true.
Assertion (A): Substituting $x = 1$ into the polynomial $f(x) = a_n x^n + \cdots + a_0$ gives $f(1) = a_n + a_{n-1} + \cdots + a_0$. If this sum is zero, then $f(1) = 0$, and $(x-1)$ is a factor. Since A is true and is directly derived from R, (A) is the correct choice.

Q8. Difference of Remainders

Answer: (A) -3
Solution: Given $f(x) = x^3 - 4x + 6$.

$$\begin{aligned} r_1 &= f(1) = (1)^3 - 4(1) + 6 = 3 \\ r_2 &= f(-2) = (-2)^3 - 4(-2) + 6 = -8 + 8 + 6 = 6 \\ r_1 - r_2 &= 3 - 6 = -3 \end{aligned}$$

Q9. Simultaneous Equations for p and q

Answer: (C) $p = 3, q = -4$
Solution: $f(2) = 0 \implies 8 + 4p + 2q - 12 = 0 \implies 2p + q = 2$ —(i)
$f(-3) = 0 \implies -27 + 9p - 3q - 12 = 0 \implies 3p - q = 13$ —(ii)
Adding (i) and (ii): $5p = 15 \implies p = 3$.
Substituting $p = 3$ into (i): $6 + q = 2 \implies q = -4$.

Q10. Non-Routine: Remainder of Composition

Answer: (B) r
Solution: The remainder of $f(x)$ divided by $(x-2)$ is $f(2) = r$.
The remainder of $f(x^2)$ divided by $(x - \sqrt{2})$ is $f((\sqrt{2})^2)$.
Since $(\sqrt{2})^2 = 2$, the remainder is $f(2)$, which is r.

Q11. Sum of Reciprocals of Roots

Answer: (B) $\frac{5}{6}$
Solution: Roots from $(x-1)(x-3)(x+2)$ are $\alpha = 1, \beta = 3, \gamma = -2$.

$$\begin{aligned}\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} &= \frac{1}{1} + \frac{1}{3} - \frac{1}{2} \\ &= \frac{6+2-3}{6} = \frac{5}{6}\end{aligned}$$

Q12. Remainder Theorem Calculation

Solution:
To find the remainder when $f(x)$ is divided by $(2x-1)$, we evaluate $f\left(\frac{1}{2}\right)$:

$$\begin{aligned}f\left(\frac{1}{2}\right) &= 3\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) - 4 \\ &= \frac{3}{8} - \frac{2}{4} + \frac{5}{2} - 4 \\ &= \frac{3-4+20-32}{8} = -\frac{13}{8}\end{aligned}$$

The remainder is $-\frac{13}{8}$.

Q13. Value of k via Factor Theorem

Solution:
Since $(x-3)$ is a factor, $g(3) = 0$:

$$\begin{aligned}2(3)^3 - 7(3)^2 + k(3) - 3 &= 0 \\ 54 - 63 + 3k - 3 &= 0 \\ 3k - 12 &= 0 \implies k = 4\end{aligned}$$

Q14. Complete Factorisation

Solution:
Dividing $h(x) = x^3 + 3x^2 - 4x - 12$ by $(x+2)$ using synthetic division:

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -4 & -12 \\ & & -2 & -2 & 12 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

The quadratic quotient is $x^2 + x - 6$. Factorising it: $x^2 + x - 6 = (x+3)(x-2)$.
Thus, $h(x) = (x+2)(x+3)(x-2)$.

Q15. Simultaneous Equations

Solution:
$p(1) = -1 \implies a + b - 4 = -1 \implies a + b = 3 \quad \ldots(1)$
$p(-1) = -11 \implies -a + b - 4 = -11 \implies -a + b = -7 \quad \ldots(2)$
Adding (1) and (2): $2b = -4 \implies b = -2$.
From (1): $a - 2 = 3 \implies a = 5$.

Q16. Factor Proof and Complete Factorisation

Solution:
$f(-\frac{1}{2}) = 2(-\frac{1}{8}) + 7(\frac{1}{4}) + 7(-\frac{1}{2}) + 2 = -\frac{1}{4} + \frac{7}{4} - \frac{7}{2} + 2 = \frac{-1+7-14+8}{4} = 0.$
Thus, $(2x+1)$ is a factor.
By division: $f(x) = (2x+1)(x^2+3x+2) = (2x+1)(x+1)(x+2)$.

Q17. Garden Bed Polynomial

Solution:

(i) $V(2) = 2^3 - 4(2)^2 + 2 + 6 = 8 - 16 + 8 = 0$. By Factor Theorem, $(x-2)$ is a factor.

(ii) $V(x) = (x-2)(x^2-2x-3) = (x-2)(x-3)(x+1)$.

(iii) For $x > 3$, dimensions are positive. Factors are $(x-2), (x-3), (x+1)$.

(iv) For $x = 5$, dimensions are $3, 2, 6$. The length is 6 m.

Q18. Box Packaging Word Problem

Solution:
Given $V(x) = 2x^3 + x^2 - 13x + 6$.

(i) $V(2) = 2(2)^3 + (2)^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$. By the Factor Theorem, $(x-2)$ is a factor.

(ii) Dividing $V(x)$ by $(x-2)$ gives $2x^2+5x-3$. Factorising the quadratic: $2x^2+6x-x-3 = (2x-1)(x+3)$.
Complete factorisation: $V(x) = (x-2)(2x-1)(x+3)$.

(iii) The roots are $x = 2, 0.5, -3$. Only $x = 2$ is a positive whole number greater than 1.

(iv) $V(3) = 2(3)^3 + (3)^2 - 13(3) + 6 = 54 + 9 - 39 + 6 = 30$. The volume is 30 cm^3.

Q19. Case-Based Polynomial Project

Solution:

(i) $P(1) = 0 \implies 1 + a + b - 8 = 0 \implies a + b = 7$.

(ii) $P(-2) = -72 \implies -8 + 4a - 2b - 8 = -72 \implies 2a - b = -28$.

(iii) Adding equations: $3a = -21 \implies a = -7$. Then $-7 + b = 7 \implies b = 14$.

(iv) $P(x) = x^3 - 7x^2 + 14x - 8 = (x-1)(x-2)(x-4)$. Roots are $1, 2, 4$ (all integers).

Q20. Mixed-Concept Proportion Question

Solution:
Given a, b, c are in continued proportion, let $\frac{a}{b} = \frac{b}{c} = k$. Thus $b = ck$ and $a = ck^2$.

(i) **LHS:** $\frac{a+b}{b+c} = \frac{ck^2+ck}{ck+c} = \frac{ck(k+1)}{c(k+1)} = k$.
RHS: $\frac{a}{b} = \frac{ck^2}{ck} = k$. Hence, proven.

(ii) Using alternendo: $\frac{a+b}{a-b} = \frac{b+c}{b-c}$.
Applying componendo and dividendo: $\frac{2a}{2b} = \frac{2b}{2c} \implies \frac{a}{b} = \frac{b}{c}$.
This is the definition of continued proportion.

(iii) $b^2 = ac = 2 \times 8 = 16 \implies b = 4.$

Evaluation: $\dfrac{2^2 + 4^2}{4^2 + 8^2} = \dfrac{20}{80} = \dfrac{1}{4}.$

(iv) Since $\dfrac{a}{b} = \dfrac{b}{c}$, then $\dfrac{a}{c} = \dfrac{a}{b} \cdot \dfrac{b}{c} = \dfrac{a}{b} \cdot \dfrac{a}{b} = \dfrac{a^2}{b^2}.$

Q21 (iv). Determining the Quantities

Solution:

Given $k = \dfrac{a}{b} = \dfrac{b}{c} = \dfrac{c}{d}.$

From $a = bk$ and $c = dk$, we get $a + c = k(b + d)$.

Substituting the given values: $10 = k(5) \implies k = 2.$

Using $b + d = 5$ and $b = ck = dk^2$:

$dk^2 + d = 5 \implies d(2^2) + d = 5 \implies 5d = 5 \implies d = 1.$

Thus: $c = dk = 2,\ b = ck = 4,\ a = bk = 8.$

The quantities are $\{8, 4, 2, 1\}$.