ICSE — Class X Mathematics

Chapter Worksheet: Linear Inequations

Model Solutions

Solutions

Section A — Multiple Choice Questions

Q1.
Correct Option: (A) $\{1, 2, 3, 4\}$

Solution:

$$\begin{aligned} 2x - 3 &< 7 \\ 2x &< 7 + 3 \\ 2x &< 10 \\ x &< 5 \end{aligned}$$

Since $x \in \mathbb{N} = \{1, 2, 3, \ldots\}$, the values less than 5 are $\{1, 2, 3, 4\}$.

Q2.
Correct Option: (B) $\{0, 1, 2, 3, \ldots\}$

Solution:

$$\begin{aligned} 5 - 3x &\leq 8 \\ -3x &\leq 8 - 5 \\ -3x &\leq 3 \end{aligned}$$

Dividing both sides by -3 and *reversing* the inequality:

$$x \geq -1$$

Since $x \in \mathbb{W} = \{0, 1, 2, 3, \ldots\}$, the values satisfying $x \geq -1$ are all whole numbers, i.e. $\{0, 1, 2, 3, \ldots\}$.

Q3.
Correct Option: (B) $\{x : x > 4,\ x \in \mathbb{R}\}$

Solution:

$$\begin{aligned} 3x + 2 &> 14 \\ 3x &> 12 \\ x &> 4 \end{aligned}$$

In set notation: $\{x : x > 4,\ x \in \mathbb{R}\}$.

Q4.
Correct Option: (B) Is reversed

Reason: Dividing (or multiplying) both sides of an inequality by a *negative* number reverses the direction of the inequality sign. For example: $6 > 4$, but dividing by -2 gives $-3 < -2$.

Q5.
Correct Option: (B) Closed circle at -2, open circle at 3, shading between them.

Reason: In $-2 \leq x < 3$:

- $x = -2$ **is included** (inequality $\leq$) $\Rightarrow$ **closed** (filled) circle at -2.
- $x = 3$ **is excluded** (strict inequality $<$) $\Rightarrow$ **open** (hollow) circle at 3.

The solution region is shaded between -2 and 3.

Q6.
Correct Option: (B) 4

Solution:

$$-3 < 2x - 1 \leq 5$$

Add 1 throughout:

$$-3 + 1 < 2x \leq 5 + 1$$
$$-2 < 2x \leq 6$$

Divide throughout by 2:

$$-1 < x \leq 3$$

For $x \in \mathbb{Z}$: values strictly greater than -1 and at most 3 are $\{0, 1, 2, 3\}$.
Number of integers $= 4$.

Q7.
Correct Option: (D) A is false, but R is true.

Verification of Assertion A:

$$x + 5 > 3$$
$$x > 3 - 5$$
$$x > -2$$

For $x \in \mathbb{Z}$, the solution is $\{\ldots, -1, 0, 1, 2, \ldots\}$ (all integers strictly greater than -2). The assertion incorrectly includes -2 and -3 in the solution set. Hence **Assertion A is false**.

Verification of Reason R: Adding or subtracting the same number on both sides of an inequality does *not* change the direction of the inequality sign. This is a valid algebraic property. Hence **Reason R is true**.

Q8.
Correct Option: (A) $-3 \leq x < 4$

Solution:

Inequation 1:

$$\begin{aligned} 2x - 5 &< 3 \\ 2x &< 8 \\ x &< 4 \end{aligned}$$

Inequation 2:

$$\begin{aligned} 3x + 1 &\geq -8 \\ 3x &\geq -9 \\ x &\geq -3 \end{aligned}$$

Intersection of $x < 4$ and $x \geq -3$:

$$-3 \leq x < 4, \quad x \in \mathbb{R}.$$

Q9.
Correct Option: (B) Incorrect, because multiplying both sides by zero is not a valid operation on an inequality.

Reason: Multiplying both sides of an inequality by zero reduces both sides to zero and yields $0 > 0$, which is a contradiction. However, this does *not* mean the original inequality has no solution; it means that multiplication by zero is an inadmissible operation on inequalities. The solution set of $x > 2$ for $x \in \mathbb{R}$ is $\{x : x > 2,\ x \in \mathbb{R}\}$, which is non-empty.

Q10.
Correct Option: (B) For $x \in \mathbb{N}$ the set is $\{1\}$; for $x \in \mathbb{W}$ the set is $\{0, 1\}$.

Solution:

$$\begin{aligned} 3x - 6 &< 0 \\ 3x &< 6 \\ x &< 2 \end{aligned}$$

- $x \in \mathbb{N} = \{1, 2, 3, \ldots\}$: values less than 2 give $\{1\}$.
- $x \in \mathbb{W} = \{0, 1, 2, \ldots\}$: values less than 2 give $\{0, 1\}$.

Section B — Short Answer Questions

Q11.

Given: $\dfrac{2x-1}{3} \geq \dfrac{x-2}{4}, \quad x \in \mathbb{Z}.$

Multiply both sides by the LCM of 3 and 4, which is 12:

$$12 \times \frac{2x-1}{3} \geq 12 \times \frac{x-2}{4}$$
$$4(2x-1) \geq 3(x-2)$$
$$8x-4 \geq 3x-6$$
$$8x-3x \geq -6+4$$
$$5x \geq -2$$
$$x \geq -\frac{2}{5}$$

Since $-\dfrac{2}{5} = -0.4$, we need $x \geq -0.4$.

For $x \in \mathbb{Z}$, the smallest integer satisfying this is 0.

Solution set: $\{x : x \geq 0,\ x \in \mathbb{Z}\} = \{0, 1, 2, 3, \ldots\}$

Q12.

Given: $\dfrac{3x}{5} - \dfrac{x-2}{3} < 1, \quad x \in \mathbb{R}.$

Multiply both sides by the LCM of 5 and 3, which is 15:

$$15 \times \frac{3x}{5} - 15 \times \frac{x-2}{3} < 15 \times 1$$
$$9x - 5(x-2) < 15$$
$$9x - 5x + 10 < 15$$
$$4x + 10 < 15$$
$$4x < 5$$
$$x < \frac{5}{4}$$

Solution in set notation: $\left\{x : x < \dfrac{5}{4},\ x \in \mathbb{R}\right\}$

Q13.

Given: $-5 \leq 2x-3 \leq 7, \quad x \in \mathbb{R}.$

Add 3 throughout the compound inequation:

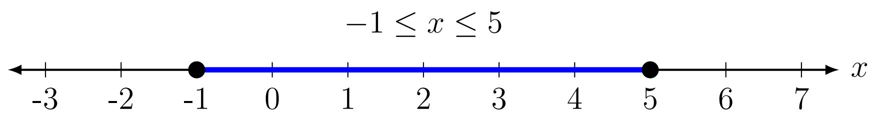
$$-5+3 \leq 2x-3+3 \leq 7+3$$
$$-2 \leq 2x \leq 10$$

Divide throughout by 2:

$$-1 \leq x \leq 5$$

Solution in set notation: $\{x : -1 \leq x \leq 5,\ x \in \mathbb{R}\}$

Number line representation:



Q14.

Given: $8 - 3x > 2x - 7, \quad x \in \mathbb{N}$.

$$8 - 3x > 2x - 7$$
$$8 + 7 > 2x + 3x$$
$$15 > 5x$$
$$x < 3$$

For $x \in \mathbb{N} = \{1, 2, 3, \ldots\}$: values less than 3 are $\{1, 2\}$.

Solution set: $\{1, 2\}$

Largest element of the solution set: 2

Q15.

Given: $2(3x - 1) \leq 4x + 6$ **and** $3(x + 2) > 2x + 5, \quad x \in \mathbb{W}$.

Inequation 1:

$$2(3x - 1) \leq 4x + 6$$
$$6x - 2 \leq 4x + 6$$
$$6x - 4x \leq 6 + 2$$
$$2x \leq 8$$
$$x \leq 4$$

Inequation 2:

$$3(x + 2) > 2x + 5$$
$$3x + 6 > 2x + 5$$
$$3x - 2x > 5 - 6$$
$$x > -1$$

Intersection: $x > -1$ **and** $x \leq 4$, i.e. $-1 < x \leq 4$.

For $x \in \mathbb{W} = \{0, 1, 2, 3, \ldots\}$: values satisfying $-1 < x \leq 4$ are $\{0, 1, 2, 3, 4\}$.

Solution set: $\{0, 1, 2, 3, 4\}$

Section C — Long Answer Questions

Q16. [Word Problem]

Given: Daily profit = $(40x - 150)$ rupees, where x is the number of units sold.
Fixed daily overhead = Rs 250.
Condition for net profit: daily profit > overhead.

Step 1: Form the inequation.

$$40x - 150 > 250$$

Step 2: Solve algebraically.

$$\begin{aligned} 40x - 150 &> 250 \\ 40x &> 250 + 150 \\ 40x &> 400 \\ x &> 10 \end{aligned}$$

Step 3: Apply domain restriction.
Since $x \in \mathbb{N}$ and $x > 10$, the smallest natural number satisfying this is $x = 11$.

The shopkeeper must sell a minimum of 11 units per day.

Q17. [Word Problem]

Given: Breadth = x m, Length = $(x + 5)$ m, $x \in \mathbb{Z}$.
Perimeter must satisfy: $46 < P \leq 66$.

Step 1: Express perimeter in terms of x.

$$\begin{aligned} P &= 2(\text{length} + \text{breadth}) \\ &= 2\big((x+5) + x\big) \\ &= 2(2x + 5) \\ &= 4x + 10 \end{aligned}$$

(i) Compound inequation:

$$46 < 4x + 10 \leq 66$$

(ii) Algebraic solution:

Subtract 10 throughout:

$$\begin{aligned} 46 - 10 < 4x &\leq 66 - 10 \\ 36 < 4x &\leq 56 \end{aligned}$$

Divide throughout by 4:

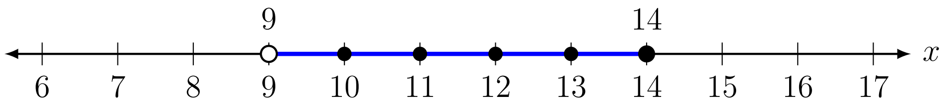
$$9 < x \leq 14$$

(iii) Solution set for $x \in \mathbb{Z}$:

Integer values strictly greater than 9 and at most 14:

$$\{10,\ 11,\ 12,\ 13,\ 14\}$$

Number line representation:



Q18. [Case-Based Question]

Given: Arjun's age $= x$ years ($x \in \mathbb{N}$); Riya's age $= (2x+3)$ years.

(i) Translating the conditions into inequations:

- **Condition I** (Arjun's age ≥ 5): $x \geq 5$
- **Condition II** (Riya's age < 25): $2x + 3 < 25$
- **Condition III** (sum of ages ≤ 35): $x + (2x+3) \leq 35$

(ii) Solving each inequation:

Condition I:

$$x \geq 5$$

Condition II:

$$\begin{aligned} 2x + 3 &< 25 \\ 2x &< 22 \\ x &< 11 \end{aligned}$$

Condition III:

$$\begin{aligned} x + 2x + 3 &\leq 35 \\ 3x + 3 &\leq 35 \\ 3x &\leq 32 \\ x &\leq \frac{32}{3} \\ x &\leq 10.\overline{6} \end{aligned}$$

Since $x \in \mathbb{N}$, this gives $x \leq 10$.

(iii) Simultaneous solution:

All three conditions:

$$x \geq 5, \quad x < 11, \quad x \leq 10.$$

The most restrictive upper bound is $x \leq 10$ (from Condition III). Combined with $x \geq 5$:

$$5 \leq x \leq 10, \quad x \in \mathbb{N}.$$

Solution set: $\{5,\ 6,\ 7,\ 8,\ 9,\ 10\}$

(iv) Greatest possible sum of their ages:

The greatest value of x in the solution set is 10.

$$\begin{aligned} \text{Arjun's age} &= 10 \text{ years} \\ \text{Riya's age} &= 2(10) + 3 = 23 \text{ years} \\ \text{Sum} &= 10 + 23 = \mathbf{33} \text{ years} \end{aligned}$$

Q19. [Mixed-Concept Question]

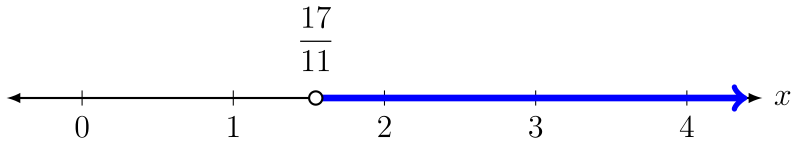
(i) *Given:* $\dfrac{5x-2}{3} - \dfrac{3x-1}{4} > 1, \quad x \in \mathbb{R}.$

Multiply both sides by the LCM of 3 and 4, which is 12:

$$\begin{aligned} 12 \times \frac{5x-2}{3} - 12 \times \frac{3x-1}{4} &> 12 \times 1 \\ 4(5x-2) - 3(3x-1) &> 12 \\ 20x - 8 - 9x + 3 &> 12 \\ 11x - 5 &> 12 \\ 11x &> 17 \\ x &> \frac{17}{11} \end{aligned}$$

Solution in set notation: $\left\{x : x > \dfrac{17}{11},\ x \in \mathbb{R}\right\}$

Number line representation:



(ii) Additional condition: $x \leq 6$.

Combined: $\dfrac{17}{11} < x \leq 6, \quad x \in \mathbb{Z}.$

Since $\dfrac{17}{11} \approx 1.545$, the integers greater than $\dfrac{17}{11}$ and at most 6 are:

$$\{2,\ 3,\ 4,\ 5,\ 6\}$$

(iii) Is $x = 3$ in the solution set for $x \in \mathbb{Z}$?

Justification:

$$x = 3 \quad \Rightarrow \quad 3 > \frac{17}{11} = 1.\overline{54}$$

Since $3 > \dfrac{17}{11}$ and $3 \leq 6$, the value $x = 3$ **satisfies** the inequation. Hence $x = 3$ **belongs** to the solution set.

Q20. [High-Difficulty Board-Level Question]

Given:

$$\frac{2x+1}{3} \geq \frac{x-2}{2} - 1 \qquad \text{and} \qquad \frac{4-3x}{5} < \frac{x+2}{3}, \quad x \in \mathbb{R}.$$

(i) Solving each inequation separately:

Inequation 1: $\dfrac{2x+1}{3} \geq \dfrac{x-2}{2} - 1$

Multiply throughout by the LCM of 3 and 2, which is 6:

$$6 \times \frac{2x+1}{3} \geq 6 \times \frac{x-2}{2} - 6 \times 1$$

$$2(2x+1) \geq 3(x-2) - 6$$

$$4x + 2 \geq 3x - 6 - 6$$

$$4x + 2 \geq 3x - 12$$

$$4x - 3x \geq -12 - 2$$

$$x \geq -14$$

Solution of Inequation 1: $\{x : x \geq -14,\ x \in \mathbb{R}\}$

Inequation 2: $\dfrac{4-3x}{5} < \dfrac{x+2}{3}$

Multiply throughout by the LCM of 5 and 3, which is 15:

$$15 \times \frac{4-3x}{5} < 15 \times \frac{x+2}{3}$$

$$3(4-3x) < 5(x+2)$$

$$12 - 9x < 5x + 10$$

$$12 - 10 < 5x + 9x$$

$$2 < 14x$$

$$x > \frac{2}{14}$$

$$x > \frac{1}{7}$$

Solution of Inequation 2: $\left\{x : x > \frac{1}{7},\ x \in \mathbb{R}\right\}$

(ii) Intersection of the two solution sets:

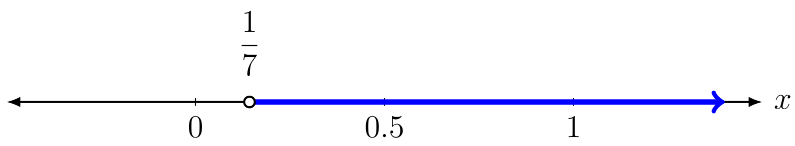
$$x \geq -14 \quad \text{and} \quad x > \frac{1}{7}$$

Since $\frac{1}{7} > -14$, the binding (more restrictive) condition is $x > \frac{1}{7}$.

Final solution:

$$\left\{x : x > \frac{1}{7},\ x \in \mathbb{R}\right\}$$

(iii) Number line representation:



(iv) Integer values in the solution set for $x \in \mathbb{Z}$:

Since $\frac{1}{7} \approx 0.143$, the integers strictly greater than $\frac{1}{7}$ begin at 1. There is no upper bound.

$$\text{Integer solution set} = \{1,\ 2,\ 3,\ 4,\ \ldots\} = \{x : x \geq 1,\ x \in \mathbb{Z}\}$$