Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (B) A circle with centre O and radius 5 cm.

Reason: The locus of all points at a constant distance from a fixed point is a **circle**. Here the fixed point is O and the constant distance is 5 cm, so the locus is a circle with centre O and radius 5 cm.

Q2. Correct Option: (C) The perpendicular bisector of the segment AB.

Reason: By the standard locus theorem, the set of all points equidistant from two fixed points A and B is the **perpendicular bisector** of segment AB. It is a complete line, not just the midpoint.

Q3. Correct Option: (C) The pair of angle bisectors of the angles formed at P.

Reason: The locus of points equidistant from two intersecting lines consists of **both** angle bisectors at the point of intersection. This gives two perpendicular lines passing through P, one bisecting the acute angles and one bisecting the obtuse angles.

Q4. Correct Option: (A) It is the locus of all points at distance 2 cm from O, and all four points P, Q, R, S lie on it.

Reason: The dashed curve is a circle of radius 2 cm centred at O. All four points P, Q, R, S are marked on the dashed circle, so each is at exactly 2 cm from O. All four points lie on the locus.

Q5. Correct Option: (C) Drawing the perpendicular bisectors of all three sides of the triangle.

Reason: The circumcentre is equidistant from all three vertices. A point equidistant from the endpoints of a side lies on the perpendicular bisector of that side. The intersection of the perpendicular bisectors of all three sides gives the circumcentre.

Q6. Correct Option: (B) Drawing the angle bisectors from each vertex.

Reason: The incentre is equidistant from all three sides. A point equidistant from two lines forming an angle lies on the angle bisector. The intersection of the three angle bisectors of the triangle gives the incentre, which is the centre of the inscribed circle.

Q7. Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Reason:

Assertion A is TRUE. Since $PA = PB = 3$ cm, point P is equidistant from A and B, so by the locus theorem it must lie on the perpendicular bisector of AB.

Reason R is TRUE and directly invokes the locus theorem (equidistant from two points $\Rightarrow$ on the perpendicular bisector) to explain why P lies on the perpendicular bisector of AB. R is the correct explanation of A.

Q8. Correct Option: (C) 2

Reason:

Locus 1: Points equidistant from A and B form the perpendicular bisector of AB, which is the vertical dashed line through the midpoint O.

Locus 2: Points at distance $r = 2$ cm from O form the dotted circle of radius 2 cm centred at O.

Since $AB = 4$ cm, half $= 2$ cm $= r$. The perpendicular bisector passes through O and intersects the circle of radius 2 cm at exactly **2 points**: P (above) and Q (below), as shown in the figure.

Q9. Correct Option: (B) The interior of a circle with centre O and radius 4 cm, excluding the boundary.

Reason: The condition is $OP < 4$ cm (strictly less than). This means P lies **inside** the circle of radius 4 cm centred at O, but **not** on the circle itself (which would require $OP = 4$ cm exactly). Hence the locus is the open interior, excluding the boundary.

Q10. Correct Option: (C) $d_1 = d_2$

Reason: The angle bisector of two intersecting lines is the locus of all points equidistant from both lines. Since point X lies on bisector b_1, its perpendicular distance from ℓ_1 equals its perpendicular distance from ℓ_2, i.e., $d_1 = d_2$.

Q11. Correct Option: (C) 2

Reason:

Condition 1: $PA = PB \Rightarrow P$ lies on the perpendicular bisector of AB, which is a straight line.

Condition 2: $PA = 5$ cm $\Rightarrow P$ lies on a circle of radius 5 cm centred at A.

$AB = 6$ cm, so the midpoint M is 3 cm from each of A and B. The perpendicular bisector passes through M perpendicularly to AB.

The perpendicular bisector intersects the circle of radius 5 cm centred at A. The distance from A to the perpendicular bisector is 3 cm. Since $3 < 5$ (the radius), the circle intersects the perpendicular bisector at **2 points**.

Verification: If P is on the perpendicular bisector at distance h from M (perpendicular distance from AB):

$$PA^2 = AM^2 + h^2 = 3^2 + h^2 = 25$$
$$h^2 = 16 \implies h = 4 \text{ cm}$$

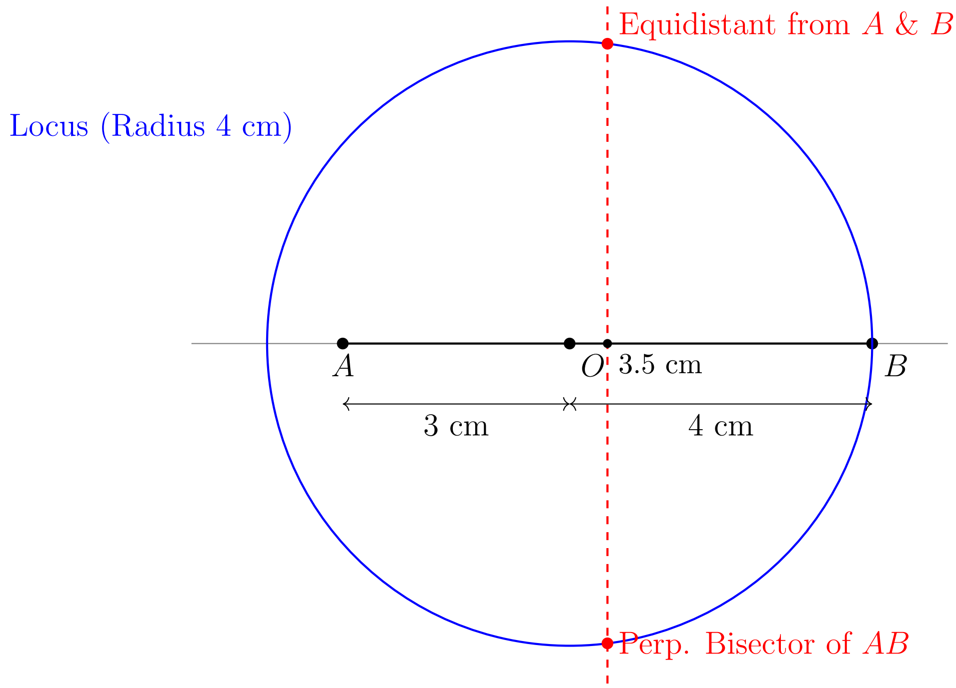
Two positions exist: one above AB and one below AB.

Q12.

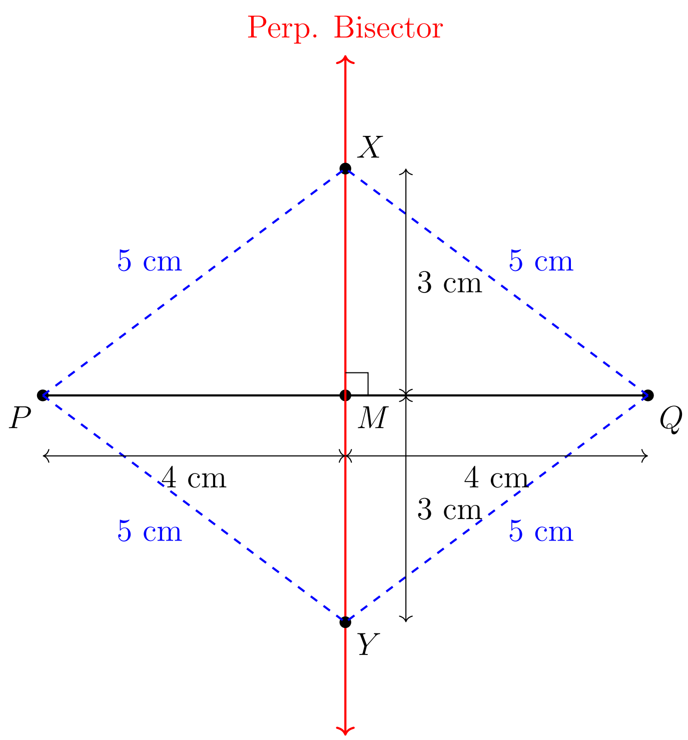
(i) A circle centered at O with a radius of 4 cm.

(ii) 1 point. (Point B lies exactly on the circle, but the other intersection falls outside the segment to the left of A).

(iii) 2 points. (The perpendicular bisector of AB is 0.5 cm from O, which is less than the 4 cm radius, so it crosses the circle twice).



Q13.



(i) **Construction:** Perpendicular bisector intersecting PQ at midpoint M.

(ii) **Verification:** By measurement (or Pythagoras: $\sqrt{3^2 + 4^2}$), $XP = XQ = 5$ cm and $YP = YQ = 5$ cm.

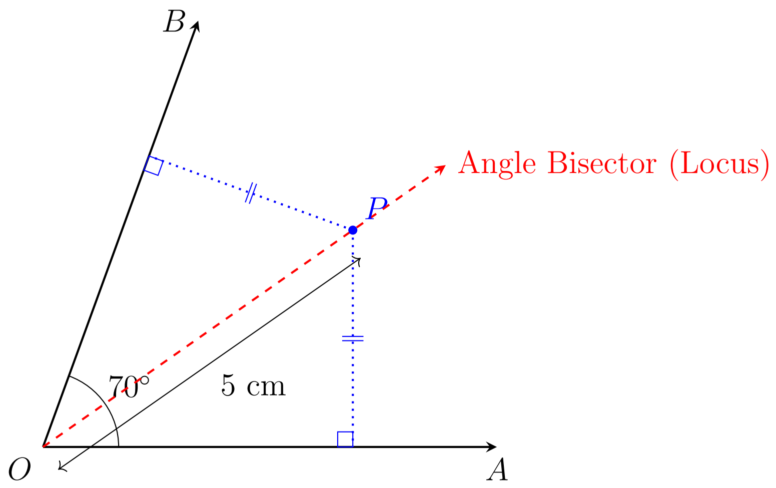
(iii) Any point on the perpendicular bisector of a segment is geometrically defined as being equidistant from the endpoints of that segment.

Q14.

(i) *Construction:* The interior angle bisector of $\angle AOB$.

(ii) *Verification:* Perpendicular distances from P to OA and OB will measure equally.

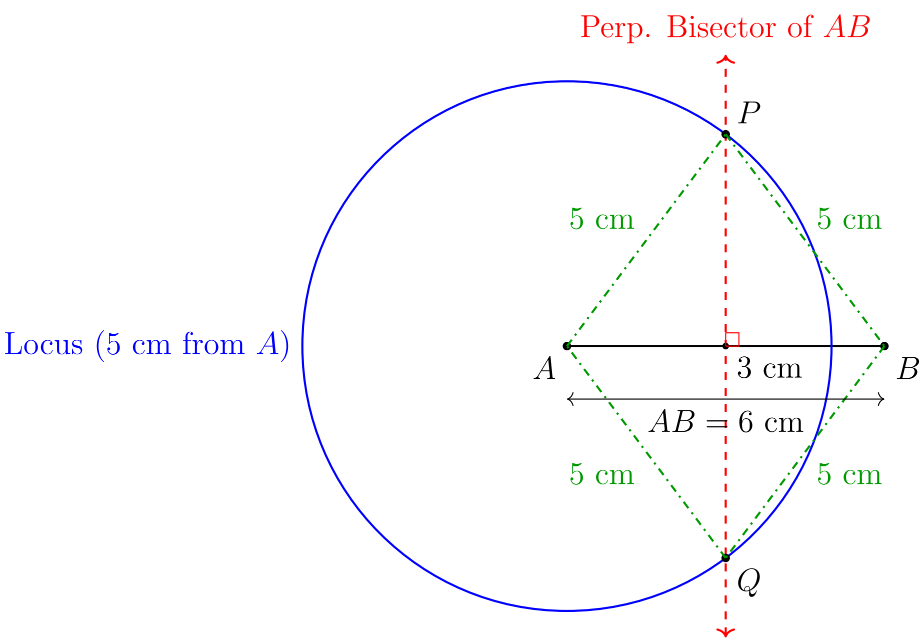
(iii) **2 distinct loci.** (If the arms are extended into intersecting straight lines, there are two angle bisectors: one for the interior angle and one for the exterior/supplementary angle. *Note: If referring strictly to rays, the answer is 1.*)



Q15.

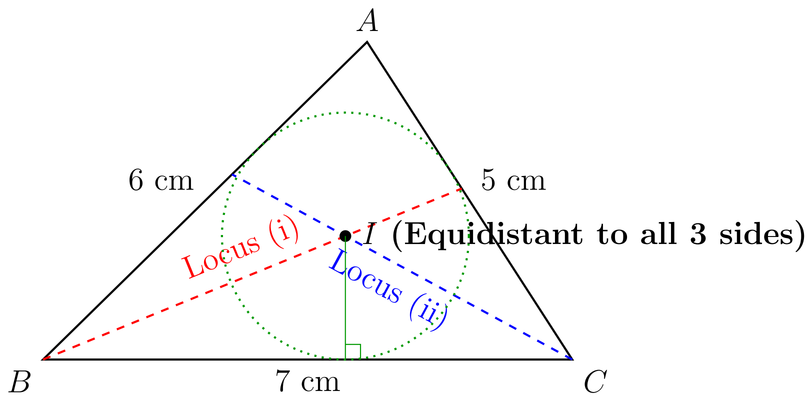
(i) & (ii) *Construction:* A perpendicular bisector of AB, and a circle of radius 5 cm centered at A.

(iii) Points P and Q form a quadrilateral $APBQ$ where all sides are 5 cm. Geometrically, these points serve as the vertices of a **rhombus**.



Q16.

- **The Triangle:** $\triangle ABC$ is constructed with sides $AB = 6$ cm, $BC = 7$ cm, and $CA = 5$ cm.
- **Locus (i):** The locus of a point equidistant from sides AB and BC is the angle bisector of $\angle B$ (shown as the red dashed line).
- **Locus (ii):** The locus of a point equidistant from sides BC and CA is the angle bisector of $\angle C$ (shown as the blue dashed line).
- **Point I:** The intersection of these two loci is a special point called the **Incentre**. Because it satisfies both conditions simultaneously, it is mathematically equidistant from all three sides of the triangle.
- **Visual Proof:** The green dotted circle centered at I represents this equal distance, perfectly touching AB, BC, and CA without crossing them.



Q17. Given: Rectangle $ABCD$, $AB = 10$ m, $BC = 8$ m; O is the centre of the garden; leash length $= 3$ m.

(i) **Locus and applicable theorem:**

The dog is tied at O with a leash of 3 m. It can move to any point that is at most 3 m from O. The **boundary** of the region the dog can reach is the set of all points **exactly** 3 m from O.

Locus: A **circle with centre O and radius 3 m**.

Region reached: The interior of this circle (including the boundary), i.e., all points within 3 m of O.

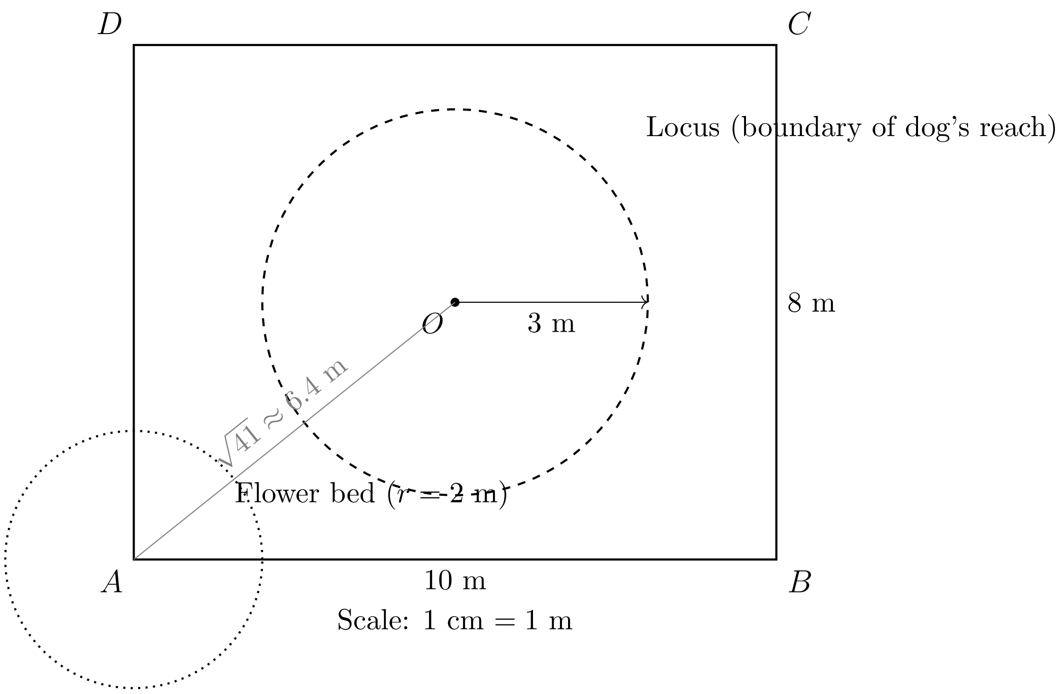
Locus Theorem applied: *The locus of a point that moves so that it remains at a constant distance from a fixed point is a circle centred at that fixed point with that constant distance as radius.*

Since O is the centre of the rectangle, its distances from the sides are:

$$\text{Distance from } AB \text{ and } CD = \frac{BC}{2} = 4 \text{ m} \qquad \text{Distance from } AD \text{ and } BC = \frac{AB}{2} = 5 \text{ m}$$

Since $3 < 4 < 5$, the entire circle of radius 3 m lies **completely inside** the garden.

(ii) **Scale drawing** (scale: 1 cm = 1 m):



Construction steps:

i. Draw rectangle $ABCD$ with $AB = 10$ cm and $BC = 8$ cm (scale 1 cm = 1 m).

ii. Mark centre O at the intersection of the diagonals (5 cm from AB, 4 cm from BC).

iii. With O as centre and radius = 3 cm, draw the circle using a compass. This is the locus (boundary of dog's reach).

(iii) **Can the dog reach the flower bed at corner A?**

The flower bed is a circle of radius 2 m centred at $A(0, 0)$. The closest point of the flower bed to O is along the line OA, at a distance of:

$$OA = \sqrt{\left(\frac{AB}{2}\right)^2 + \left(\frac{BC}{2}\right)^2} = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41} \approx 6.40 \text{ m}$$

The nearest point of the flower bed from O is:

Minimum distance $= OA -$ radius of flower bed $= \sqrt{41} - 2 \approx 6.40 - 2 = 4.40$ m

Since 4.40 m $>$ 3 m (leash length), the dog **cannot reach** any part of the flower bed.

On the scale drawing: The locus circle (radius 3 cm) and the flower bed circle (radius 2 cm at corner A) do not overlap or touch — they are clearly separated. ✓

(iv) **Leash extended to 7 m — new locus and corners:**

New locus: A circle with centre O and radius 7 m.

Changes in the locus:

- The radius increases from 3 m to 7 m.
- The distances from O to the sides are 4 m and 5 m, both less than 7 m. Therefore the new circle **extends outside the rectangle** and is partially clipped by the garden walls.

Can the dog reach the corners?

Distance from O to each corner:

$$OA = OB = OC = OD = \sqrt{5^2 + 4^2} = \sqrt{41} \approx 6.40 \text{ m}$$

Since 6.40 m $<$ 7 m (new leash length), the dog **can reach all four corners** of the garden when the leash is extended to 7 m.

Square $ABCD$, side 6 cm; O is the centre.

Q18. (a) (i) **Locus Theorem:** The locus of all points in a plane that are at a fixed distance from a fixed point is a circle with the fixed point as its center and the fixed distance as its radius.

Geometric Figure: Circle.

(Drawn as the dashed blue circle L_3 in the diagram below).

(ii) **Locus Theorem:** The locus of all points in a plane that are equidistant from two intersecting straight lines consists of the pair of perpendicular angle bisectors of the angles formed by those intersecting lines.

Geometric Figures: A pair of intersecting straight lines / angle bisectors.

(Drawn as the dashed red lines L_1 and L_2 in the diagram below).

(iii) The mobile tower T must satisfy both conditions simultaneously, meaning its position must lie on the intersection of the circle (L_3) and the angle bisectors (L_1 and L_2). A circle centered at the intersection of two lines will intersect those lines at exactly 4 points.

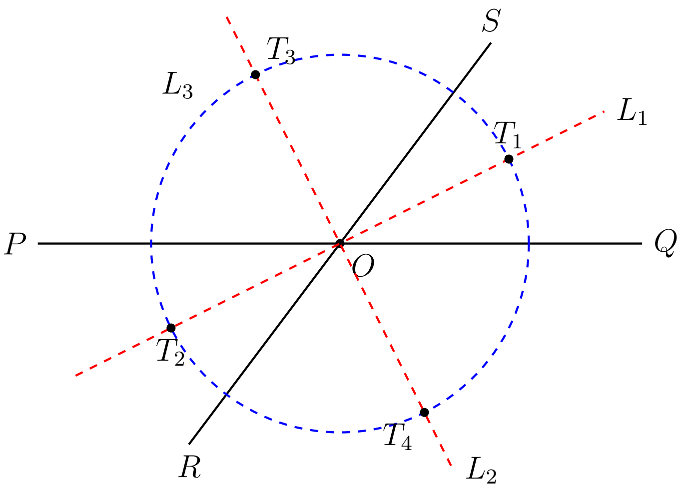
Number of positions: 4 possible positions.

(Marked as T_1, T_2, T_3, T_4 in the diagram below).

(iv) **Number of valid positions:** 1 valid position (T_1).

Reason: The problem restricts the installation to the interior of $\angle SOQ$. Inside this single angle, there is only one angle bisector ray extending from O. This single ray will intersect the circular locus (L_3) at exactly one point.

Completed Diagram:



Q19. (i) **Locus equidistant from X and Y:**
Draw $XY = 7$ cm. With compass radius > 3.5 cm, draw arcs from X and Y (above and below). Join intersections to get perpendicular bisector ℓ. Label midpoint M ($XM = YM = 3.5$ cm).
Locus: Line ℓ (perpendicular bisector of XY).

(ii) **Locus of Q at distance 3 cm from Z:**
Mark Z on XY with $XZ = 2$ cm. With centre Z and radius 3 cm, draw a full circle. This circle is the locus of Q.

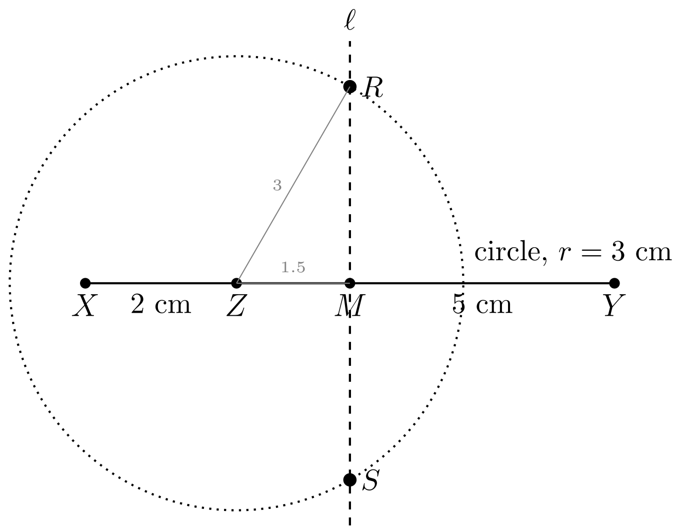
(iii) **Points R and S satisfying both loci:**
The required points lie on **both** ℓ and the circle centred at Z.
Distance from Z to ℓ:

$$ZM = XM - XZ = 3.5 - 2 = 1.5 \text{ cm}$$

Since $ZM = 1.5 < 3$ (radius), the line ℓ intersects the circle at **2 points**.
Height of each point above/below XY:

$$h = \sqrt{3^2 - 1.5^2} = \sqrt{9 - 2.25} = \sqrt{6.75} = \frac{3\sqrt{3}}{2} \approx 2.60 \text{ cm}$$

Label these intersection points R (above XY) and S (below XY).
Diagram:



(iv) **Locus satisfying both conditions:**
The locus of points equidistant from X and Y is ℓ (perpendicular bisector of XY). The locus of points at distance 3 cm from Z is the circle centred at Z.
The required points are the **intersections of ℓ and the circle**, which are points R and S.

$$\boxed{\text{Number of such points} = \mathbf{2}}$$

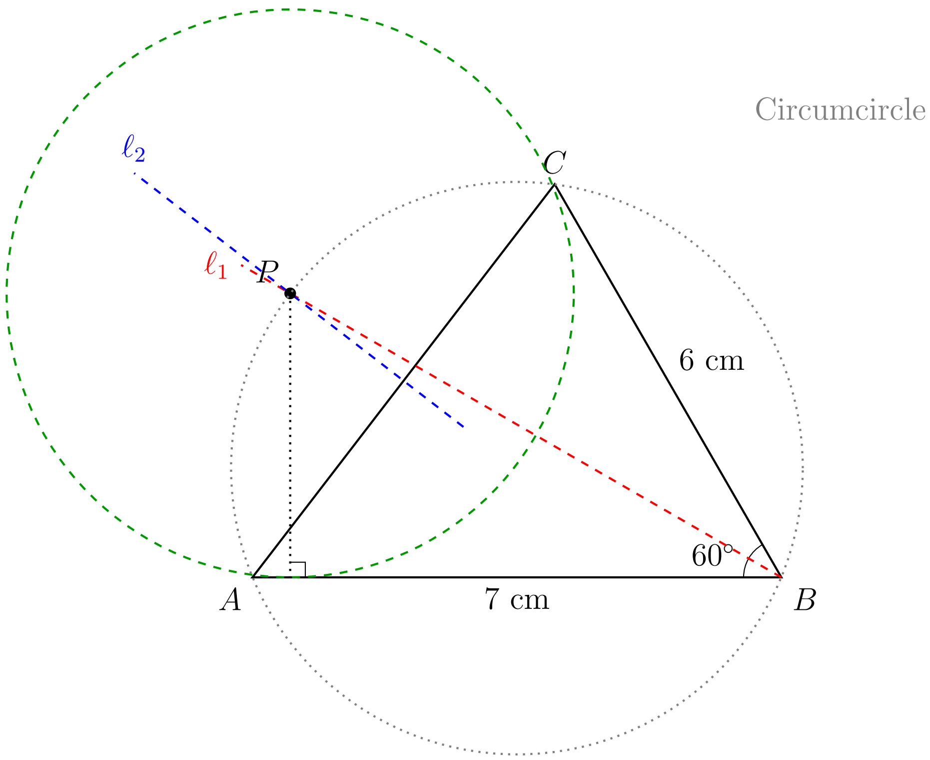
Since $ZM = 1.5$ cm $<$ radius $= 3$ cm, the perpendicular bisector cuts the circle at exactly 2 points, visible clearly on the diagram.

Q20. Construct $\triangle ABC$ with $AB = 7$ cm, $BC = 6$ cm, $\angle ABC = 60°$.

(i) **Locus ℓ_1 — equidistant from sides AB and BC:**
ℓ_1 is the **bisector of** $\angle ABC$. By the angle bisector locus theorem, every point on ℓ_1 is equidistant from lines AB and BC. Since $\angle ABC = 60°$, ℓ_1 makes $30°$ with each side. Extend ℓ_1 beyond the triangle.

(ii) **Locus ℓ_2 — equidistant from A and C:**
ℓ_2 is the **perpendicular bisector of** AC. Construct it by drawing arcs of equal radius from A and C and joining the intersection points.

(iii) **Point P on both ℓ_1 and ℓ_2:**
Mark P at the intersection of ℓ_1 (angle bisector of $\angle ABC$) and ℓ_2 (perpendicular bisector of AC).
Diagram:



(iv) **Circle with centre P, radius = perpendicular distance from P to AB:**

Draw a perpendicular from P to line AB; let its foot be F. Set compass to PF and draw the circle.

(a) **What the circle represents:**
Since P lies on ℓ_1 (angle bisector of $\angle ABC$), P is equidistant from sides AB, BC. The perpendicular distance from P to AB equals the perpendicular distance from P to BC. This circle is the **incircle of** $\triangle ABC$ — it is tangent to all three sides.

(b) **Does the circle pass through vertex B?**
No. P lies on the angle bisector ℓ_1, so it is equidistant from lines AB and BC (not from the vertices). The radius $= PF$ is the perpendicular distance to side AB, which is less than PB. Hence B lies **outside** the incircle.