Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (C) Row matrix

Reason: A matrix with exactly one row and more than one column is, by definition, called a **row matrix**.

Q2. Correct Option: (C) O

Reason:

$$3I - 3I = 3(I - I) = 3 \cdot O = O$$

Subtracting any matrix from itself yields the null matrix O.

Q3. Correct Option: (C) A matrix of order 2×2 **and a matrix of order** 2×2

Reason: Matrix addition is defined **only** when both matrices have the **same order**. Among the given options, only option (C) pairs two matrices of identical order 2×2.

Q4. Correct Option: (B) 1

Reason:

$$A + B = \begin{pmatrix} 3 & -1 \\ 0 & 4 \end{pmatrix} + \begin{pmatrix} -2 & 5 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 3 + (-2) & -1 + 5 \\ 0 + 1 & 4 + (-3) \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}$$

The element in the **second row, first column** of $A + B$ is **1**.

Q5. Correct Option: (A) $\begin{pmatrix} -1 & \frac{3}{2} \\ -\frac{5}{2} & 0 \end{pmatrix}$

Reason:

$$-\frac{1}{2}M = -\frac{1}{2}\begin{pmatrix} 2 & -3 \\ 5 & 0 \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{2} \times 2 & -\dfrac{1}{2} \times (-3) \\ -\dfrac{1}{2} \times 5 & -\dfrac{1}{2} \times 0 \end{pmatrix} = \begin{pmatrix} -1 & \dfrac{3}{2} \\ -\dfrac{5}{2} & 0 \end{pmatrix}$$

Q6. Correct Option: (B) 2×2

Reason: For the product PQ to exist, the number of columns of P must equal the number of rows of Q.

$$P_{2\times 3} \times Q_{3\times 2} \implies PQ \text{ has order } 2 \times 2.$$

(Outer dimensions of the compatible pair give the order of the product.)

Q7. Correct Option: (B) Both A and R are true, but R is not the correct explanation of A.

Reason:

- **Assertion (A) is TRUE.** Matrix multiplication is generally not commutative; $AB \neq BA$ for arbitrary square matrices. (A standard counterexample suffices.)

- **Reason (R) is TRUE.** Matrix multiplication PQ requires the number of columns of P to equal the number of rows of Q.
- However, **R does not explain A.** The compatibility condition (R) is about when multiplication is *defined*, not about why it is *non-commutative*. Non-commutativity arises from the nature of the row-by-column summation, which is a separate property. Hence R is not the correct explanation of A.

Q8. Correct Option: (D) O (the null matrix)

Reason: The identity matrix I satisfies $AI = A$ and $IA = A$ for any square matrix A of the same order.

$$A \cdot I - I \cdot A = A - A = O$$

Q9. Correct Option: (C) 8

Reason: Equating corresponding elements:

$$\begin{aligned} x + 2 = 5 &\implies x = 3 \\ y - 1 = 4 &\implies y = 5 \end{aligned}$$

$$\therefore \quad x + y = 3 + 5 = \mathbf{8}$$

Q10. Correct Option: (A) $\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

Reason: From $2X + A = B$:

$$2X = B - A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$$

$$X = \frac{1}{2}\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{2} & -\dfrac{1}{2} \\ -\dfrac{1}{2} & -\dfrac{1}{2} \end{pmatrix}$$

Solutions

Section B — Short Answer Questions

Q11. The total number of elements in a matrix of order $m \times n$ is $m \times n$. We need all factor pairs (m, n) such that $m \times n = 8$.

Order	**Product**	**Type**
1×8	$1 \times 8 = 8$	Row matrix (1 row, more than 1 column)
8×1	$8 \times 1 = 8$	Column matrix (more than 1 row, 1 column)
2×4	$2 \times 4 = 8$	Neither
4×2	$4 \times 2 = 8$	Neither

Thus there are **four** possible orders: $1 \times 8,\ 8 \times 1,\ 2 \times 4,\ 4 \times 2$.

Q12. We compute $2P - Q + R$ step by step.

Step 1: Compute $2P$.

$$2P = 2\begin{pmatrix} 5 & -2 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} 10 & -4 \\ 6 & 14 \end{pmatrix}$$

Step 2: Compute $2P - Q$.

$$2P - Q = \begin{pmatrix} 10 & -4 \\ 6 & 14 \end{pmatrix} - \begin{pmatrix} -1 & 4 \\ 6 & -3 \end{pmatrix} = \begin{pmatrix} 10 - (-1) & -4 - 4 \\ 6 - 6 & 14 - (-3) \end{pmatrix} = \begin{pmatrix} 11 & -8 \\ 0 & 17 \end{pmatrix}$$

Step 3: Compute $2P - Q + R$.

$$2P - Q + R = \begin{pmatrix} 11 & -8 \\ 0 & 17 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} 11 + 2 & -8 + 0 \\ 0 + (-4) & 17 + 1 \end{pmatrix}$$

$$\boxed{2P - Q + R = \begin{pmatrix} 13 & -8 \\ -4 & 18 \end{pmatrix}}$$

Q13. Given $A = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

Part 1: Verify $A + O = O + A = A$.

$$A + O = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 3+0 & -2+0 \\ 1+0 & 4+0 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix} = A \quad \checkmark$$

$$O + A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 0+3 & 0+(-2) \\ 0+1 & 0+4 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix} = A \quad \checkmark$$

Hence $A + O = O + A = A$. (The null matrix is the additive identity.)

Part 2: Verify $A \cdot I = I \cdot A = A$.

$$\begin{aligned} A \cdot I &= \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 3 \cdot 1 + (-2) \cdot 0 & 3 \cdot 0 + (-2) \cdot 1 \\ 1 \cdot 1 + 4 \cdot 0 & 1 \cdot 0 + 4 \cdot 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix} = A \quad \checkmark \end{aligned}$$

$$\begin{aligned} I \cdot A &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 1 \cdot 3 + 0 \cdot 1 & 1 \cdot (-2) + 0 \cdot 4 \\ 0 \cdot 3 + 1 \cdot 1 & 0 \cdot (-2) + 1 \cdot 4 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix} = A \quad \checkmark \end{aligned}$$

Hence $A \cdot I = I \cdot A = A$. (The identity matrix is the multiplicative identity.)

Q14. We are given:

$$3X - 2\begin{pmatrix}1 & -3\\ 4 & 0\end{pmatrix} = \begin{pmatrix}7 & 3\\ 2 & 6\end{pmatrix}$$

Step 1: Compute the scalar product.

$$2\begin{pmatrix}1 & -3\\ 4 & 0\end{pmatrix} = \begin{pmatrix}2 & -6\\ 8 & 0\end{pmatrix}$$

Step 2: Isolate $3X$.

$$3X = \begin{pmatrix}7 & 3\\ 2 & 6\end{pmatrix} + \begin{pmatrix}2 & -6\\ 8 & 0\end{pmatrix} = \begin{pmatrix}9 & -3\\ 10 & 6\end{pmatrix}$$

Step 3: Solve for X.

$$X = \frac{1}{3}\begin{pmatrix}9 & -3\\ 10 & 6\end{pmatrix}$$

$$\boxed{X = \begin{pmatrix}3 & -1\\ \dfrac{10}{3} & 2\end{pmatrix}}$$

Q15. *Rule:* The product $A_{m\times n} \times B_{p\times q}$ is defined if and only if $n = p$. The order of the resulting matrix is $m \times q$.

(i) $A_{2\times3} \times B_{3\times4}$:
Number of columns of $A = 3 =$ number of rows of $B = 3$.
Defined. Order of $AB = 2 \times 4$.

(ii) $C_{3\times2} \times D_{3\times2}$:
Number of columns of $C = 2 \neq$ number of rows of $D = 3$.
Not defined.

(iii) $E_{1\times2} \times F_{2\times1}$:
Number of columns of $E = 2 =$ number of rows of $F = 2$.
Defined. Order of $EF = 1 \times 1$.

(iv) $G_{2\times2} \times H_{2\times3}$:
Number of columns of $G = 2 =$ number of rows of $H = 2$.
Defined. Order of $GH = 2 \times 3$.

Q16. Given $A = \begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}$ and $B = \begin{pmatrix}0 & -2\\ 4 & 1\end{pmatrix}$.

Step 1: Compute AB.

$$\begin{aligned}AB &= \begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}\begin{pmatrix}0 & -2\\ 4 & 1\end{pmatrix}\\ &= \begin{pmatrix}2(0)+1(4) & 2(-2)+1(1)\\ (-1)(0)+3(4) & (-1)(-2)+3(1)\end{pmatrix}\\ &= \begin{pmatrix}0+4 & -4+1\\ 0+12 & 2+3\end{pmatrix} = \begin{pmatrix}4 & -3\\ 12 & 5\end{pmatrix}\end{aligned}$$

Step 2: Compute BA.

$$BA = \begin{pmatrix} 0 & -2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0(2) + (-2)(-1) & 0(1) + (-2)(3) \\ 4(2) + 1(-1) & 4(1) + 1(3) \end{pmatrix}$$

$$= \begin{pmatrix} 0+2 & 0-6 \\ 8-1 & 4+3 \end{pmatrix} = \begin{pmatrix} 2 & -6 \\ 7 & 7 \end{pmatrix}$$

Verification:

$$AB = \begin{pmatrix} 4 & -3 \\ 12 & 5 \end{pmatrix} \neq \begin{pmatrix} 2 & -6 \\ 7 & 7 \end{pmatrix} = BA$$

Since the $(1,1)$ entries alone differ $(4 \neq 2)$, we conclude $AB \neq BA$, confirming that matrix multiplication is **not commutative** in general. ✓

Solutions

Section C — Long Answer Questions

Q17. Given:

$$A = \begin{pmatrix} 72 & 68 & 80 \\ 55 & 60 & 74 \end{pmatrix}, \qquad B = \begin{pmatrix} 65 & 70 & 75 \\ 50 & 62 & 70 \end{pmatrix}$$

(Rows: Asha, Bimal   Columns: Test 1, Test 2, Test 3)

(i) **Matrix $A + B$:**

$$A + B = \begin{pmatrix} 72+65 & 68+70 & 80+75 \\ 55+50 & 60+62 & 74+70 \end{pmatrix} = \begin{pmatrix} 137 & 138 & 155 \\ 105 & 122 & 144 \end{pmatrix}$$

Each element represents the **combined marks** of each student (across Mathematics and Science) in each test.

(ii) **Matrix $A - B$:**

$$A - B = \begin{pmatrix} 72-65 & 68-70 & 80-75 \\ 55-50 & 60-62 & 74-70 \end{pmatrix} = \begin{pmatrix} 7 & -2 & 5 \\ 5 & -2 & 4 \end{pmatrix}$$

Interpretation: Each element represents how many marks more (or less) each student scored in Mathematics compared to Science in that test. A positive value means better performance in Mathematics; a negative value means better performance in Science. For example, -2 in position $(1,2)$ means Asha scored 2 marks less in Mathematics than in Science in Test 2.

(iii) **Matrix C for Chetan:**

Chetan scores exactly half the sum of Asha's and Bimal's marks, i.e.,

$$C = \frac{1}{2}(A + B) = \frac{1}{2}\begin{pmatrix} 137 & 138 & 155 \\ 105 & 122 & 144 \end{pmatrix}$$

$$\boxed{C = \begin{pmatrix} 68.5 & 69 & 77.5 \\ 52.5 & 61 & 72 \end{pmatrix}}$$

(Row 1: Mathematics; Row 2: Science)

(iv) **Higher total in Test 2:**
From the matrix $A + B$, the second column gives the combined marks in Test 2:

$$\text{Asha's total in Test 2} = 138$$
$$\text{Bimal's total in Test 2} = 122$$

Since $138 > 122$, **Asha** has a higher total in Test 2 across both subjects.

Q18. Given:

$$F = \begin{pmatrix} 120 & 80 \\ 90 & 150 \end{pmatrix} \quad \text{(Rows: Factory } P, Q; \text{ Columns: Products } X, Y\text{)}$$

(i) **New production matrix F' after 10% increase in Factory P:**
Factory P (row 1) is scaled by 1.10; Factory Q (row 2) is unchanged.

$$F'_P = 1.10 \times \begin{pmatrix} 120 & 80 \end{pmatrix} = \begin{pmatrix} 132 & 88 \end{pmatrix}$$

$$\boxed{F' = \begin{pmatrix} 132 & 88 \\ 90 & 150 \end{pmatrix}}$$

(ii) **Revenue matrix $F \cdot S$:**

$$F \cdot S = \begin{pmatrix} 120 & 80 \\ 90 & 150 \end{pmatrix} \begin{pmatrix} 50 \\ 40 \end{pmatrix}$$
$$= \begin{pmatrix} 120 \times 50 + 80 \times 40 \\ 90 \times 50 + 150 \times 40 \end{pmatrix} = \begin{pmatrix} 6000 + 3200 \\ 4500 + 6000 \end{pmatrix} = \begin{pmatrix} 9200 \\ 10500 \end{pmatrix}$$

Interpretation:

- Rs 9200: Total weekly revenue of **Factory** P.
- Rs 10500: Total weekly revenue of **Factory** Q.

(iii) **Profit matrix using $F \cdot (S - C_0)$:**
Unit cost matrix:

$$C_0 = \begin{pmatrix} 30 \\ 30 \end{pmatrix}$$

$$S - C_0 = \begin{pmatrix} 50 \\ 40 \end{pmatrix} - \begin{pmatrix} 30 \\ 30 \end{pmatrix} = \begin{pmatrix} 20 \\ 10 \end{pmatrix}$$

$$F \cdot (S - C_0) = \begin{pmatrix} 120 & 80 \\ 90 & 150 \end{pmatrix} \begin{pmatrix} 20 \\ 10 \end{pmatrix}$$
$$= \begin{pmatrix} 120 \times 20 + 80 \times 10 \\ 90 \times 20 + 150 \times 10 \end{pmatrix} = \begin{pmatrix} 2400 + 800 \\ 1800 + 1500 \end{pmatrix} = \begin{pmatrix} 3200 \\ 3300 \end{pmatrix}$$

Interpretation: Weekly profit of Factory P is **Rs** 3200 and of Factory Q is **Rs** 3300.

Q19. Given:

$$M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(i) **Compute $M^2 = M \times M$:**

$$M^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1(1)+1(0) & 1(1)+1(1) \\ 0(1)+1(0) & 0(1)+1(1) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

(ii) **Compute $M^3 = M^2 \times M$:**

$$M^3 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1(1)+2(0) & 1(1)+2(1) \\ 0(1)+1(0) & 0(1)+1(1) \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

(iii) **Pattern for M^n:**
Observing the results:

$$M^1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad M^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \quad M^3 = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

The $(1,2)$ entry equals n in each case, while all other entries remain constant. Thus, for any positive integer n:

$$\boxed{M^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}}$$

(iv) **Verification:**
Check $n = 1$:

$$M^1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = M \quad \checkmark$$

Check $n = 2$:

$$M^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

This matches the result computed directly in part (i). ✓

Q20. Given:

$$2\begin{pmatrix} x & 5 \\ 3 & y \end{pmatrix} + 3\begin{pmatrix} 1 & -2 \\ a & 4 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 15 & 26 \end{pmatrix}$$

(i) **LHS after scalar multiplication:**

$$2\begin{pmatrix} x & 5 \\ 3 & y \end{pmatrix} = \begin{pmatrix} 2x & 10 \\ 6 & 2y \end{pmatrix}$$

$$3\begin{pmatrix} 1 & -2 \\ a & 4 \end{pmatrix} = \begin{pmatrix} 3 & -6 \\ 3a & 12 \end{pmatrix}$$

Adding the two matrices:

$$\begin{pmatrix} 2x & 10 \\ 6 & 2y \end{pmatrix} + \begin{pmatrix} 3 & -6 \\ 3a & 12 \end{pmatrix} = \begin{pmatrix} 2x+3 & 4 \\ 6+3a & 2y+12 \end{pmatrix}$$

(ii) **Four equations by equating corresponding elements:**

$$\begin{aligned}
(1,1):&\quad 2x+3=7\\
(1,2):&\quad 4=4 \quad \text{(automatically satisfied)}\\
(2,1):&\quad 6+3a=15\\
(2,2):&\quad 2y+12=26
\end{aligned}$$

(iii) **Solving for x, y, and a:**

$$2x+3=7 \implies 2x=4 \implies \boxed{x=2}$$

$$6+3a=15 \implies 3a=9 \implies \boxed{a=3}$$

$$2y+12=26 \implies 2y=14 \implies \boxed{y=7}$$

(iv) **The two matrices with values substituted:**

$$\text{First matrix:} \quad \begin{pmatrix} x & 5 \\ 3 & y \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix}$$

$$\text{Second matrix:} \quad \begin{pmatrix} 1 & -2 \\ a & 4 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$$

Verification:

$$2\begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix} + 3\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ 6 & 14 \end{pmatrix} + \begin{pmatrix} 3 & -6 \\ 9 & 12 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 15 & 26 \end{pmatrix} \quad \checkmark$$

Q21. Given: $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\quad B = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$

(i) **Compute AB and BA:**

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix} = \begin{pmatrix} p+0 & 0+2q \\ 3p+0 & 0+4q \end{pmatrix} = \begin{pmatrix} p & 2q \\ 3p & 4q \end{pmatrix}$$

$$BA = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} p+0 & 2p+0 \\ 0+3q & 0+4q \end{pmatrix} = \begin{pmatrix} p & 2p \\ 3q & 4q \end{pmatrix}$$

(ii) **Condition $AB = BA$ implies $p = q$. Finding p and q given $AB = 2A$:**
Setting $AB = BA$:

$$\begin{pmatrix} p & 2q \\ 3p & 4q \end{pmatrix} = \begin{pmatrix} p & 2p \\ 3q & 4q \end{pmatrix}$$

Equating the $(1,2)$ entries: $2q = 2p \implies p = q$.
Equating the $(2,1)$ entries: $3p = 3q \implies p = q$. $\checkmark$
Hence $AB = BA$ holds if and only if $\mathbf{p} = \mathbf{q}$.

Now, using $AB = 2A$ with $p = q$:

$$AB = \begin{pmatrix} p & 2p \\ 3p & 4p \end{pmatrix}, \qquad 2A = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$$

Equating $(1,1)$ entries: $p = 2$.
Checking all entries with $p = 2$:

$$AB = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} = 2A \quad \checkmark$$

$$\therefore \quad \boxed{p = q = 2}$$

(iii) **Find $A^2 - (p+q)A$:**
With $p = q = 2$, we have $p + q = 4$.

$$A^2 = A \times A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1(1) + 2(3) & 1(2) + 2(4) \\ 3(1) + 4(3) & 3(2) + 4(4) \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}$$

$$(p+q)A = 4 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 12 & 16 \end{pmatrix}$$

$$A^2 - (p+q)A = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} - \begin{pmatrix} 4 & 8 \\ 12 & 16 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 3 & 6 \end{pmatrix}$$

(iv) **Find k such that $A^2 - kA = 2I$, and verify:**

$$A^2 - kA = 2I \implies \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} - k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$
$$\begin{pmatrix} 7 - k & 10 - 2k \\ 15 - 3k & 22 - 4k \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Equating corresponding elements:

$$\begin{aligned} (1,1): &\quad 7 - k = 2 \implies k = 5 \\ (1,2): &\quad 10 - 2k = 0 \implies k = 5 \quad \checkmark \\ (2,1): &\quad 15 - 3k = 0 \implies k = 5 \quad \checkmark \\ (2,2): &\quad 22 - 4k = 2 \implies k = 5 \quad \checkmark \end{aligned}$$

$$\therefore \quad \boxed{k = 5}$$

Verification:

$$A^2 - 5A = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} - 5 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I \quad \checkmark$$