Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (A) 440 **cm**^{2}

Reason: CSA of cylinder $= 2\pi rh$:

$$\text{CSA} = 2 \times \frac{22}{7} \times 7 \times 10 = 2 \times 22 \times 10 = \mathbf{440}\ \text{cm}^2$$

Q2. Correct Option: (B) 528 **cm**^{3}

Reason: Volume of cone $= \frac{1}{3}\pi r^2 h$:

$$V = \frac{1}{3} \times \frac{22}{7} \times 6^2 \times 14 = \frac{1}{3} \times \frac{22}{7} \times 36 \times 14 = \frac{1}{3} \times 22 \times 36 \times 2 = \frac{1{,}584}{3} = \mathbf{792}\ \text{cm}^3$$

Q3. Correct Option: (B) 616 **cm**^{2}

Reason: Diameter $= 14$ cm, so $r = 7$ cm. TSA of sphere $= 4\pi r^2$:

$$\text{TSA} = 4 \times \frac{22}{7} \times 7^2 = 4 \times \frac{22}{7} \times 49 = 4 \times 22 \times 7 = \mathbf{616}\ \text{cm}^2$$

Q4. Correct Option: (A) 17 **cm**

Reason: Using the Pythagorean theorem: $l = \sqrt{r^2 + h^2}$:

$$l = \sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = \mathbf{17}\ \text{cm}$$

Q5. Correct Option: (C) 8 **cm**

Reason: Volume is conserved when melted and recast. Let height of cylinder $= h$.

$$\begin{aligned} \frac{4}{3}\pi r_s^3 &= \pi r_c^2 h \\ \frac{4}{3} \times 6^3 &= 6^2 \times h \\ \frac{4}{3} \times 216 &= 36h \\ 288 &= 36h \\ h &= \mathbf{8}\ \text{cm} \end{aligned}$$

Q6. Correct Option: (B) Rs 1,760

Reason: CSA of cylinder $= 2\pi rh$:

$$\text{CSA} = 2 \times \frac{22}{7} \times 3.5 \times 10 = 2 \times 22 \times 5 = 220\ \text{m}^2$$

$$\text{Cost} = 220 \times 8 = \textbf{Rs}\ 1{,}760$$

Q7. Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Reason:

Assertion A is TRUE:

$$\frac{\text{Volume of cone}}{\text{Volume of cylinder}} = \frac{\frac{1}{3}\pi r^2 h}{\pi r^2 h} = \frac{1}{3}$$

Hence the volume of the cone is one-third that of the cylinder.

Reason R is TRUE and directly establishes A by citing the correct formulae and computing the ratio. R is the correct explanation of A.

Q8. Correct Option: (A) 594 **cm**^{3}

Reason: Volume of metal $= \pi(R^2 - r^2)h$, where $R = 5$ cm, $r = 4$ cm, $h = 21$ cm:

$$\begin{aligned} V &= \frac{22}{7} \times (5^2 - 4^2) \times 21 \\ &= \frac{22}{7} \times (25 - 16) \times 21 \\ &= \frac{22}{7} \times 9 \times 21 \\ &= 22 \times 9 \times 3 = \mathbf{594}\ \text{cm}^3 \end{aligned}$$

Q9. Correct Option: (A) 1540 **cm**^{3}

Reason: $r = 7$ cm, cone height $= 16$ cm.

Volume of hemisphere:

$$V_{\text{hemi}} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7^3 = \frac{2}{3} \times \frac{22}{7} \times 343 = \frac{2}{3} \times 22 \times 49 = \frac{2{,}156}{3} \approx 718.67\ \text{cm}^3$$

Volume of cone:

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 49 \times 16 = \frac{1}{3} \times 22 \times 7 \times 16 = \frac{2{,}464}{3} \approx 821.33\ \text{cm}^3$$

Total volume:

$$V = \frac{2{,}156}{3} + \frac{2{,}464}{3} = \frac{4{,}620}{3} = \mathbf{1{,}540}\ \text{cm}^3$$

Q10. Correct Option: (C) 27

Reason: Number of small spheres $= \dfrac{\text{Volume of large sphere}}{\text{Volume of small sphere}}$:

$$N = \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \left(\frac{R}{r}\right)^3 = \left(\frac{9}{3}\right)^3 = 3^3 = \mathbf{27}$$

Q11. Correct Option: (D) πr^2

Reason: When a solid hemisphere of radius r is placed on a flat surface, the flat circular face (base) is in contact with the table. The area of this circular base is:

$$\text{Area} = \pi r^2$$

(This is simply the area of a circle of radius r, not the curved surface area of the hemisphere.)

Section B — Short Answer Questions

Q12. Given: $r = 7$ cm, $h = 20$ cm. Use $\pi = \dfrac{22}{7}$.

(i) **Curved Surface Area:**

$$\text{CSA} = 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 20 = 2 \times 22 \times 20 = \mathbf{880\ cm}^2$$

(ii) **Total Surface Area:**

$$\text{TSA} = 2\pi r(h + r) = 2 \times \frac{22}{7} \times 7 \times (20 + 7) = 2 \times 22 \times 27 = \mathbf{1{,}188\ cm}^2$$

(iii) **Cost of painting:**

$$\text{Cost} = \text{TSA} \times \text{Rate} = 1{,}188 \times 5 = \textbf{Rs } 5{,}940$$

Q13. Given: Base diameter $= 24$ cm $\Rightarrow r = 12$ cm; slant height $l = 13$ cm.

(i) **Vertical height of the cone:**

$$h = \sqrt{l^2 - r^2} = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = \mathbf{5\ cm}$$

(ii) **Curved Surface Area:**

$$\text{CSA} = \pi rl = \frac{22}{7} \times 12 \times 13 = \frac{3{,}432}{7} = \mathbf{490\tfrac{2}{7}} \approx \mathbf{490.29}\ \text{cm}^{\mathbf{2}}$$

(iii) **Volume of the cone:**

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 144 \times 5 = \frac{1}{3} \times \frac{22 \times 720}{7} = \frac{15{,}840}{21} = \mathbf{754\tfrac{2}{7}} \approx \mathbf{754.29}\ \text{cm}^{\mathbf{3}}$$

Q14. Given: Inner radius $r = 6$ cm; Outer radius $R = 7$ cm.

(i) **Volume of metal:**

$$\begin{aligned} V &= \frac{4}{3}\pi(R^3 - r^3) = \frac{4}{3} \times \frac{22}{7} \times (7^3 - 6^3) \\ &= \frac{4}{3} \times \frac{22}{7} \times (343 - 216) = \frac{4}{3} \times \frac{22}{7} \times 127 \\ &= \frac{4 \times 22 \times 127}{21} = \frac{11{,}176}{21} = \mathbf{532\tfrac{4}{21}} \approx \mathbf{532.19}\ \text{cm}^{\mathbf{3}} \end{aligned}$$

(ii) **Outer curved surface area:**

$$\text{Outer CSA} = 4\pi R^2 = 4 \times \frac{22}{7} \times 7^2 = 4 \times \frac{22}{7} \times 49 = 4 \times 22 \times 7 = \mathbf{616\ cm^2}$$

Q15. Given: Sphere diameter $= 21$ cm $\Rightarrow r_s = \dfrac{21}{2}$ cm.

Cone diameter $= 7$ cm $\Rightarrow r_c = \dfrac{7}{2}$ cm; $h_c = 3$ cm.

Volume of the metallic sphere:

$$V_s = \frac{4}{3}\pi r_s^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{9{,}261}{8}$$
$$= \frac{4 \times 22 \times 9{,}261}{3 \times 7 \times 8} = \frac{8{,}14{,}968}{168} = 4{,}851 \text{ cm}^3$$

Volume of one small cone:

$$V_c = \frac{1}{3}\pi r_c^2 h_c = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 3 = \frac{22}{7} \times \frac{49}{4} = \frac{22 \times 7}{4} = \frac{154}{4} = 38.5 \text{ cm}^3$$

Number of cones:

$$N = \frac{V_s}{V_c} = \frac{4{,}851}{38.5} = \mathbf{126}$$

Q16. Given: $r = 7$ cm, $h_{\text{cylinder}} = 10$ cm; hemisphere of same radius on top.

Outer surfaces:

- CSA of cylinder (lateral surface)
- Base circle of cylinder (bottom)
- Curved surface area of hemisphere (top)

(The flat circular top of the cylinder is covered by the hemisphere and excluded.)

CSA of cylinder:

$$\text{CSA}_{\text{cyl}} = 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 10 = 440 \text{ cm}^2$$

Base area of cylinder:

$$\text{Base} = \pi r^2 = \frac{22}{7} \times 49 = 154 \text{ cm}^2$$

CSA of hemisphere:

$$\text{CSA}_{\text{hemi}} = 2\pi r^2 = 2 \times \frac{22}{7} \times 49 = 308 \text{ cm}^2$$

Total Surface Area:

$$\text{TSA} = 440 + 154 + 308 = \mathbf{902\ cm^2}$$

Section C — Long Answer Questions

Q17. Given: Cylinder: $r = 7$ cm, $h = 15$ cm; Cone: $r = 7$ cm, $h_c = 24$ cm.

(i) **Slant height of the cone:**

$$l = \sqrt{r^2 + h_c^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = \mathbf{25\ cm}$$

(ii) **Total surface area of the toy:**
Outer surfaces: CSA of cone + CSA of cylinder + base of cylinder.

$$\text{CSA of cone} = \pi r l = \frac{22}{7} \times 7 \times 25 = 550\ \text{cm}^2$$
$$\text{CSA of cylinder} = 2\pi r h = 2 \times \frac{22}{7} \times 7 \times 15 = 660\ \text{cm}^2$$
$$\text{Base of cylinder} = \pi r^2 = \frac{22}{7} \times 49 = 154\ \text{cm}^2$$
$$\text{TSA} = 550 + 660 + 154 = \mathbf{1{,}364\ cm^2}$$

(iii) **Total volume of the toy:**

$$V_{\text{cyl}} = \pi r^2 h = \frac{22}{7} \times 49 \times 15 = 2{,}310\ \text{cm}^3$$
$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h_c = \frac{1}{3} \times \frac{22}{7} \times 49 \times 24 = \frac{1}{3} \times 22 \times 7 \times 24 = 1{,}232\ \text{cm}^3$$
$$V_{\text{total}} = 2{,}310 + 1{,}232 = \mathbf{3{,}542\ cm^3}$$

(iv) **Cost of painting:**

$$\text{Cost} = 1{,}364 \times 0.05 = \textbf{Rs}\ 68.20$$

Q18. Given: External diameter $= 28$ cm $\Rightarrow R = 14$ cm; Length $= 3.5$ m $= 350$ cm; Thickness $= 7$ cm.

(i) **Internal radius:**

$$r = R - \text{thickness} = 14 - 7 = \mathbf{7\ cm}$$

(ii) **Internal volume (water capacity):**

$$V_{\text{internal}} = \pi r^2 h = \frac{22}{7} \times 49 \times 350 = 22 \times 7 \times 350 = 53{,}900\ \text{cm}^3$$
$$= \frac{53{,}900}{1{,}000} = \mathbf{53.9\ litres}$$

(iii) **Volume of iron:**

$$V_{\text{iron}} = \pi(R^2 - r^2)h = \frac{22}{7} \times (196 - 49) \times 350$$
$$= \frac{22}{7} \times 147 \times 350 = 22 \times 21 \times 350 = \mathbf{1{,}61{,}700\ cm^3}$$

(iv) **Cost of iron:**

$$\text{Mass} = V \times \text{density} = 1{,}61{,}700 \times 8 \text{ g} = 12{,}93{,}600 \text{ g} = 1{,}293.6 \text{ kg}$$
$$\text{Cost} = 1{,}293.6 \times 50 = \textbf{Rs } 64{,}680$$

Q19. Given: Hemisphere + Cylinder + Cone; $r = 21$ cm; $h_{\text{cyl}} = 20$ cm; $h_{\text{cone}} = 28$ cm.

(i) **Total volume:**

$$V_{\text{hemi}} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 9{,}261 = \frac{2}{3} \times 22 \times 1{,}323 = 19{,}404 \text{ cm}^3$$
$$V_{\text{cyl}} = \pi r^2 h = \frac{22}{7} \times 441 \times 20 = 22 \times 63 \times 20 = 27{,}720 \text{ cm}^3$$
$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h_c = \frac{1}{3} \times \frac{22}{7} \times 441 \times 28 = \frac{1}{3} \times 22 \times 63 \times 4 \times \frac{7}{7} = 12{,}936 \text{ cm}^3$$
$$V_{\text{total}} = 19{,}404 + 27{,}720 + 12{,}936 = \mathbf{60{,}060 \text{ cm}^3}$$

(ii) **Slant height and total surface area:**

$$l = \sqrt{21^2 + 28^2} = \sqrt{441 + 784} = \sqrt{1{,}225} = 35 \text{ cm}$$

Outer surfaces (flat circular interfaces are interior):

$$\text{CSA of hemisphere} = 2\pi r^2 = 2 \times \frac{22}{7} \times 441 = 2{,}772 \text{ cm}^2$$
$$\text{CSA of cylinder} = 2\pi r h = 2 \times \frac{22}{7} \times 21 \times 20 = 2 \times 22 \times 60 = 2{,}640 \text{ cm}^2$$
$$\text{CSA of cone} = \pi r l = \frac{22}{7} \times 21 \times 35 = 22 \times 3 \times 35 = 2{,}310 \text{ cm}^2$$
$$\text{TSA} = 2{,}772 + 2{,}640 + 2{,}310 = \mathbf{7{,}722 \text{ cm}^2}$$

(iii) **Mass of wooden block:**

$$\text{Mass} = V \times \rho = 60{,}060 \times 0.8 \text{ g} = 48{,}048 \text{ g} = \mathbf{48.048 \text{ kg}}$$

(iv) **Total painting cost:**

$$\text{Cost} = 7{,}722 \times 0.10 = \textbf{Rs } 772.20$$

Q20. (i) **Cone melted to sphere** ($r_{\text{cone}} = 12$ cm, $h = 6$ cm):

$$V_{\text{cone}} = \frac{1}{3}\pi \times 144 \times 6 = 288\pi$$
$$\frac{4}{3}\pi R^3 = 288\pi$$
$$R^3 = \frac{288 \times 3}{4} = 216$$
$$R = \sqrt[3]{216} = \mathbf{6 \text{ cm}}$$

(ii) **Height of conical vessel** ($r = 14$ cm, cylinder height $= 10$ cm):

$$V_{\text{cylinder}} = \pi \times 196 \times 10 = 1{,}960\pi$$

$$V_{\text{cone}} = \frac{1}{3}\pi \times 196 \times H = 1{,}960\pi$$

$$H = \frac{1{,}960 \times 3}{196} = \textbf{30 cm}$$

(iii) **Combined solid** (hemisphere $r = 21$ cm, cone on top; total height $= 49$ cm):

$$h_{\text{cone}} = 49 - 21 = 28 \text{ cm}$$

Total volume:

$$V_{\text{hemi}} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 9{,}261 = 19{,}404 \text{ cm}^3$$

$$V_{\text{cone}} = \frac{1}{3} \times \frac{22}{7} \times 441 \times 28 = 12{,}936 \text{ cm}^3$$

$$V_{\text{total}} = 19{,}404 + 12{,}936 = \textbf{32,340 cm}^3$$

Total surface area:

$$l = \sqrt{21^2 + 28^2} = \sqrt{1{,}225} = 35 \text{ cm}$$

$$\text{TSA} = 2\pi r^2 + \pi r l = \pi r(2r + l)$$

$$= \frac{22}{7} \times 21 \times (42 + 35) = 22 \times 3 \times 77 = \textbf{5,082 cm}^2$$

Q21. (i) **Volume of water in cylinder (before submersion):**

$$V_{\text{water}} = \pi r^2 H$$

(ii) **Volume of the cone:**

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 H$$

(iii) **Volume of water that overflows:**
When the cone is fully submerged in the completely filled cylinder, the volume of water displaced equals the volume of the cone, and all displaced water overflows (since the cylinder was already full).

$$V_{\text{overflow}} = V_{\text{cone}} = \frac{1}{3}\pi r^2 H$$

(iv) With $r = 7$ cm, $H = 24$ cm:

(a) **Volume of water that overflows:**

$$V_{\text{overflow}} = \frac{1}{3}\pi r^2 H = \frac{1}{3} \times \frac{22}{7} \times 49 \times 24 = \frac{1}{3} \times 22 \times 7 \times 24 = \textbf{1,232 cm}^3$$

(b) **Curved surface area of the cone:**

$$l = \sqrt{r^2 + H^2} = \sqrt{49 + 576} = \sqrt{625} = 25 \text{ cm}$$

$$\text{CSA} = \pi r l = \frac{22}{7} \times 7 \times 25 = 22 \times 25 = \mathbf{550\ cm}^2$$

(c) **Total surface area of the cylinder:**

$$\text{TSA} = 2\pi r(H + r) = 2 \times \frac{22}{7} \times 7 \times (24 + 7) = 2 \times 22 \times 31 = \mathbf{1{,}364\ cm}^2$$