Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (B) $\{1, 2, 3, 4, 5, 6\}$

Reason: The sample space S of an experiment is the set of all possible outcomes. A standard six-sided die has faces numbered 1 to 6, so:

$$S = \{1, 2, 3, 4, 5, 6\}, \qquad n(S) = 6$$

Q2. Correct Option: (D) $\frac{5}{3}$

Reason: The probability of any event E must satisfy:

$$0 \leq P(E) \leq 1$$

Since $\frac{5}{3} \approx 1.67 > 1$, it cannot be a valid probability. All other options lie within $[0, 1]$.

Q3. Correct Option: (A) $\frac{1}{2}$

Reason: Total number of balls $= 5 + 3 + 2 = 10$.

Number of balls that are **not** red $= 3 + 2 = 5$.

$$P(\text{not red}) = \frac{5}{10} = \frac{1}{2}$$

Q4. Correct Option: (C) $\frac{4}{52}$

Reason: There are 4 kings in a standard deck of 52 cards (one per suit).

$$P(\text{king}) = \frac{4}{52} = \frac{1}{13}$$

Note: Options (C) and (D) are evaluated as follows: $\frac{4}{52} = \frac{1}{13}$ is correct; $\frac{1}{4}$ is incorrect. Hence option **(C)** is the correct expression for this probability.

Q5. Correct Option: (A) $\frac{1}{2}$

Reason: The spinner has 8 equally likely outcomes: $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

Prime numbers in this set: $\{2, 3, 5, 7\}$ — there are 4 primes.

$$P(\text{prime}) = \frac{4}{8} = \frac{1}{2}$$

Q6. Correct Option: (B) $\frac{1}{10}$

Reason: Two-digit numbers range from 10 to 99.

Total number of two-digit numbers $= 99 - 10 + 1 = 90$.

Multiples of 9 in this range: $\{18, 27, 36, 45, 54, 63, 72, 81, 90\}$ — there are 9 such numbers.

$$P(\text{multiple of } 9) = \frac{9}{90} = \frac{1}{10}$$

Note: Option (D) $\frac{9}{90}$ is the unsimplified form of $\frac{1}{10}$; both are numerically equal, but option (B) is the simplified standard form.

Q7. Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Reason:

Assertion A: For a fair coin, $P(\text{tail}) = \frac{1}{2}$. Using the complementary event rule:

$$P(\text{head}) = 1 - P(\text{tail}) = 1 - \frac{1}{2} = \frac{1}{2}$$

Assertion A is TRUE.

Reason R: The law $P(E) + P(\overline{E}) = 1$ is a fundamental theorem of probability. **R is TRUE.**

R directly explains A: since tail and head are complementary events and $P(E) + P(\overline{E}) = 1$, knowing $P(\text{tail}) = \frac{1}{2}$ immediately gives $P(\text{head}) = \frac{1}{2}$. Hence **R is the correct explanation of A.**

Q8. Correct Option: (B) Event of drawing a red ball: probability $= 1$ (certain event); event of drawing a blue ball: probability $= 0$ (impossible event).

Reason: Since the bag contains **only** red balls:

$$P(\text{red ball}) = \frac{\text{number of red balls}}{\text{total balls}} = 1 \quad (\text{certain event})$$

$$P(\text{blue ball}) = \frac{0}{\text{total balls}} = 0 \quad (\text{impossible event})$$

Q9. Correct Option: (C) $\frac{3}{4}$

Reason: Sample space when two fair coins are tossed:

$$S = \{\text{HH, HT, TH, TT}\}, \qquad n(S) = 4$$

Event of getting **at least one head**: $\{\text{HH, HT, TH}\}$ — 3 favourable outcomes.

$$P(\text{at least one head}) = \frac{3}{4}$$

Alternatively: $P(\text{at least one head}) = 1 - P(\text{no head}) = 1 - \frac{1}{4} = \frac{3}{4}$.

Q10. Correct Option: (B) $\frac{2}{11}$

Reason: The word **MATHEMATICS** has 11 letters:

$$\text{M, A, T, H, E, M, A, T, I, C, S}$$

Total number of cards = 11.

The letter **M** appears **2** times (positions 1 and 6).

$$P(\text{card bears M}) = \frac{2}{11}$$

Q11. Correct Option: (B) $\frac{3}{5}$

Reason: Total number of trials = 200; event E occurred 80 times.

$$P(E) = \frac{80}{200} = \frac{2}{5}$$

Using the complementary event rule:

$$P(\overline{E}) = 1 - P(E) = 1 - \frac{2}{5} = \frac{3}{5}$$

Section B — Short Answer Questions

Q12. (i) **Complete sample space S:**

A coin gives {H, T} and a die gives {1, 2, 3, 4, 5, 6}. The sample space is:

$$S = \{(\text{H}, 1), (\text{H}, 2), (\text{H}, 3), (\text{H}, 4), (\text{H}, 5), (\text{H}, 6), \\ (\text{T}, 1), (\text{T}, 2), (\text{T}, 3), (\text{T}, 4), (\text{T}, 5), (\text{T}, 6)\}$$

(ii) **Total number of outcomes:**

$$n(S) = 2 \times 6 = 12$$

(iii) **Probability of obtaining a head and an even number:**

Favourable outcomes (Head with an even number):

$$\{(\text{H}, 2), (\text{H}, 4), (\text{H}, 6)\}$$

Number of favourable outcomes = 3.

$$P(\text{Head and even number}) = \frac{3}{12} = \frac{1}{4}$$

Q13. Total number of cards = 52.

(i) **Probability of drawing a red queen:**

There are 2 red queens in a deck (Queen of Hearts and Queen of Diamonds).

$$P(\text{red queen}) = \frac{2}{52} = \frac{1}{26}$$

(ii) **Probability of drawing a face card:**
Face cards are Jacks, Queens, and Kings. There are 3 face cards per suit $\times$ 4 suits $=$ 12 face cards.

$$P(\text{face card}) = \frac{12}{52} = \frac{3}{13}$$

(iii) **Probability of drawing neither a spade nor a heart:**
Cards that are spades or hearts $= 13 + 13 = 26$.
Cards that are **neither** a spade nor a heart (i.e., clubs and diamonds) $= 52 - 26 = 26$.

$$P(\text{neither spade nor heart}) = \frac{26}{52} = \frac{1}{2}$$

Q14. Total students $= 40$. Let M = like Mathematics, Sc = like Science.

$$n(M) = 16, \quad n(Sc) = 12, \quad n(M \cap Sc) = 12$$

(i) **Number of students who like neither subject:**
Using the union formula:

$$\begin{aligned} n(M \cup Sc) &= n(M) + n(Sc) - n(M \cap Sc) \\ &= 16 + 12 - 12 = 16 \end{aligned}$$

$$\text{Students who like neither} = 40 - n(M \cup Sc) = 40 - 16 = \mathbf{24}$$

(ii) **Probability of choosing a student who likes neither subject:**

$$P(\text{neither}) = \frac{24}{40} = \frac{3}{5}$$

(iii) **Probability of choosing a student who likes at least one subject:**

$$P(\text{at least one}) = 1 - P(\text{neither}) = 1 - \frac{3}{5} = \frac{2}{5}$$

Alternatively: $P(\text{at least one}) = \dfrac{n(M \cup Sc)}{40} = \dfrac{16}{40} = \dfrac{2}{5}$ ✓

Q15. Sample space: $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$.

(i) **Probability of a number greater than 4:**
Favourable outcomes: $\{5, 6\}$, count $= 2$.

$$P(\text{greater than 4}) = \frac{2}{6} = \frac{1}{3}$$

(ii) **Probability of a number that is both even and greater than 2:**
Even numbers greater than 2: $\{4, 6\}$, count $= 2$.

$$P(\text{even and greater than 2}) = \frac{2}{6} = \frac{1}{3}$$

(iii) **Probability of a perfect square:**
Perfect squares in S: $\{1, 4\}$ (since $1 = 1^2$ and $4 = 2^2$), count $= 2$.

$$P(\text{perfect square}) = \frac{2}{6} = \frac{1}{3}$$

Q16. Contents of the box: 4 white, 6 black, 5 yellow marbles.

Total marbles $= 4 + 6 + 5 = 15$.

(i) **Probability of drawing a white marble:**

$$P(\text{white}) = \frac{4}{15}$$

(ii) **Probability of drawing a marble that is not black:**
Number of non-black marbles $= 4 + 5 = 9$.

$$P(\text{not black}) = \frac{9}{15} = \frac{3}{5}$$

Alternatively: $P(\text{not black}) = 1 - P(\text{black}) = 1 - \frac{6}{15} = 1 - \frac{2}{5} = \frac{3}{5}$ ✓

(iii) **Probability of drawing a white or yellow marble:**
Number of white or yellow marbles $= 4 + 5 = 9$.

$$P(\text{white or yellow}) = \frac{9}{15} = \frac{3}{5}$$

Section C — Long Answer Questions

Q17. Total coins $= 8 + 5 + 4 + 3 = 20$.

(i) **Probability of Rs 1 coin:**

$$P(\text{Rs } 1) = \frac{8}{20} = \frac{2}{5}$$

(ii) **Probability of Rs 5 or Rs 10 coin:**
Favourable coins $= 4 + 3 = 7$.

$$P(\text{Rs 5 or Rs 10}) = \frac{7}{20}$$

(iii) **Probability of coin not of denomination Rs 2:**
Coins of Rs $2 = 5$. Coins not of Rs $2 = 20 - 5 = 15$.

$$P(\text{not Rs } 2) = \frac{15}{20} = \frac{3}{4}$$

Alternatively: $P(\text{not Rs } 2) = 1 - \frac{5}{20} = 1 - \frac{1}{4} = \frac{3}{4}$ ✓

(iv) **Verification of Aarav's claim:**

Coins worth more than Rs 1: Rs 2 (5 coins), Rs 5 (4 coins), Rs 10 (3 coins).
Total $= 5 + 4 + 3 = 12$ coins.

$$P(\text{worth more than Rs 1}) = \frac{12}{20} = \frac{3}{5}$$

Aarav claims the probability is **greater than** $\frac{3}{5}$. Since $\frac{12}{20} = \frac{3}{5}$, the probability is **equal to** $\frac{3}{5}$, not greater. Therefore Aarav's claim is **incorrect**.

Q18. Total students surveyed $= 120$.

Season	Number of students
Summer	35
Monsoon	42
Winter	28
Spring	15
Total	120

(i) **Probability that favourite season is Monsoon:**

$$P(\text{Monsoon}) = \frac{42}{120} = \frac{7}{20}$$

(ii) **Probability that favourite season is not Winter:**

$$P(\text{not Winter}) = 1 - P(\text{Winter}) = 1 - \frac{28}{120} = \frac{92}{120} = \frac{23}{30}$$

(iii) **Probability of Summer or Spring:**

Favourable students $= 35 + 15 = 50$.

$$P(\text{Summer or Spring}) = \frac{50}{120} = \frac{5}{12}$$

(iv) **Probability of a season whose name has more than 6 letters:**

Counting letters: Summer (6), Monsoon (7), Winter (6), Spring (6).

Only **Monsoon** has more than 6 letters (7 letters).

$$P(\text{more than 6 letters}) = \frac{42}{120} = \frac{7}{20}$$

Q19. Total tokens $= 30$. Sample space $= \{1, 2, 3, \ldots, 30\}$.

(i) **Favourable outcomes for each event:**

$$\begin{aligned}
\text{Event A (multiples of 3):} \quad & \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\} \implies n(A) = 10 \\
\text{Event B (perfect squares):} \quad & \{1, 4, 9, 16, 25\} \implies n(B) = 5 \\
\text{Event C (greater than 20):} \quad & \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30\} \implies n(C) = 10 \\
\text{Event D (prime numbers):} \quad & \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\} \implies n(D) = 10
\end{aligned}$$

(ii) **Probability of each event:**

$$P(A) = \frac{10}{30} = \frac{1}{3}$$
$$P(B) = \frac{5}{30} = \frac{1}{6}$$
$$P(C) = \frac{10}{30} = \frac{1}{3}$$
$$P(D) = \frac{10}{30} = \frac{1}{3}$$

(iii) **Event A and C (multiple of 3 AND greater than 20):**
From Event A: $\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$.
Intersecting with Event C (greater than 20):

$$A \cap C = \{21,\ 24,\ 27,\ 30\} \implies n(A \cap C) = 4$$

$$P(\text{prize}) = P(A \cap C) = \frac{4}{30} = \frac{2}{15}$$

(iv) **Event A or B (multiple of 3 OR perfect square):**
$A = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$
$B = \{1, 4, 9, 16, 25\}$
$A \cap B = \{9\}$ (common element)
$A \cup B$ (no repetition):

$$\{1, 3, 4, 6, 9, 12, 15, 16, 18, 21, 24, 25, 27, 30\} \implies n(A \cup B) = 14$$

Verification using addition rule: $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 10 + 5 - 1 = 14$ ✓

$$P(A \cup B) = \frac{14}{30} = \frac{7}{15}$$

Q20. (i) **Sample space S and $n(S)$:**

$$S = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15,\ 16,\ 17,\ 18,\ 19,\ 20\}$$

$$n(S) = 20$$

(ii) (a) **Probability of an odd number:**
Odd numbers: $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$, count $= 10$.

$$P(\text{odd}) = \frac{10}{20} = \frac{1}{2}$$

(b) **Probability of a number divisible by both 2 and 3 (i.e., divisible by 6):**
Multiples of 6 up to 20: $\{6,\ 12,\ 18\}$, count $= 3$.

$$P(\text{divisible by 6}) = \frac{3}{20}$$

(iii) **Events E and F:**
Event E (multiple of 4): $\{4, 8, 12, 16, 20\}$, $n(E) = 5$.

$$P(E) = \frac{5}{20} = \frac{1}{4}$$

Event F (less than 9): $\{1, 2, 3, 4, 5, 6, 7, 8\}$, $n(F) = 8$.

$$P(F) = \frac{8}{20} = \frac{2}{5}$$

Both E and F simultaneously (multiple of 4 AND less than 9):

$$E \cap F = \{4, 8\}, \quad n(E \cap F) = 2$$

$$P(E \cap F) = \frac{2}{20} = \frac{1}{10}$$

(iv) **Second card divisible by 4, given first card was 8:**
After removing card 8, remaining cards $= 19$.
Multiples of 4 in $\{1, \ldots, 20\}$: $\{4, 8, 12, 16, 20\}$.
After removing 8, multiples of 4 remaining $= \{4, 12, 16, 20\}$, count $= 4$.

$$P(\text{second card divisible by 4}) = \frac{4}{19}$$

Q21. Two fair dice thrown; outcomes recorded as (a, b).

(i) **Total number of outcomes:**

$$n(S) = 6 \times 6 = 36$$

(ii) **Probability that $a + b$ is:**

(a) **Equal to 7:**
Favourable outcomes: $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$, count $= 6$.

$$P(a + b = 7) = \frac{6}{36} = \frac{1}{6}$$

(b) **A prime number:**
Possible sums range from 2 to 12. Prime sums: 2, 3, 5, 7, 11.

$$\begin{aligned}
\text{Sum } &= 2: \quad (1,1) \implies 1 \text{ outcome} \\
\text{Sum } &= 3: \quad (1,2), (2,1) \implies 2 \text{ outcomes} \\
\text{Sum } &= 5: \quad (1,4), (2,3), (3,2), (4,1) \implies 4 \text{ outcomes} \\
\text{Sum } &= 7: \quad (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) \implies 6 \text{ outcomes} \\
\text{Sum } &= 11: \quad (5,6), (6,5) \implies 2 \text{ outcomes}
\end{aligned}$$

Total favourable $= 1 + 2 + 4 + 6 + 2 = 15$.

$$P(\text{sum is prime}) = \frac{15}{36} = \frac{5}{12}$$

(c) **At least 10:**
Sums ≥ 10: 10, 11, 12.

$$\begin{aligned} \text{Sum } &= 10: \quad (4,6),(5,5),(6,4) \implies 3 \text{ outcomes} \\ \text{Sum } &= 11: \quad (5,6),(6,5) \implies 2 \text{ outcomes} \\ \text{Sum } &= 12: \quad (6,6) \implies 1 \text{ outcome} \end{aligned}$$

Total favourable $= 3 + 2 + 1 = 6$.

$$P(a+b \geq 10) = \frac{6}{36} = \frac{1}{6}$$

(iii) **Probability that $a \times b$ is even:**
Using the complementary event: the product is **odd** only when **both** a and b are odd.
Odd numbers on a die: $\{1, 3, 5\}$, count $= 3$.

$$P(\text{both odd}) = \frac{3}{6} \times \frac{3}{6} = \frac{9}{36} = \frac{1}{4}$$

$$P(\text{product even}) = 1 - P(\text{product odd}) = 1 - \frac{1}{4} = \frac{3}{4}$$

(iv) **Probability that $|a - b| = 2$:**
Systematically listing all favourable outcomes:

$$\begin{aligned} a - b = 2: &\quad (3,1),(4,2),(5,3),(6,4) \implies 4 \text{ outcomes} \\ b - a = 2: &\quad (1,3),(2,4),(3,5),(4,6) \implies 4 \text{ outcomes} \end{aligned}$$

Total favourable outcomes $= 4 + 4 = 8$.

$$P(|a-b| = 2) = \frac{8}{36} = \frac{2}{9}$$