ICSE MATHEMATICS WORKSHEET — COMPLETE SOLUTIONS

Chapter: Quadratic Equations in One Variable

Solutions

Section A

Q1. Correct Option: (a) $k < 3$

For real and distinct roots, the discriminant must be strictly positive:

$$\begin{aligned} \Delta &> 0 \\ (-6)^2 - 4(k)(3) &> 0 \\ 36 - 12k &> 0 \\ k &< 3 \end{aligned}$$

Q2. Correct Option: (c)

Simplify option (c):

$$\begin{aligned} x(2x+3) &= x^2 + 1 \\ 2x^2 + 3x &= x^2 + 1 \\ x^2 + 3x - 1 &= 0 \end{aligned}$$

This is a valid quadratic equation (degree 2, one variable).
Option (b): $(x+1)^2 = x^2 - 4 \Rightarrow 2x + 5 = 0$ (linear).
Option (d): contains $\sqrt{x}$ (not a polynomial).
Option (a): becomes degree 4 on simplification.

Q3. Correct Option: (c) is correct.

From the graph, the signs are determined as follows:

- Parabola opens **downward** $\Rightarrow a < 0$.
- The y-intercept $f(0) = c$ lies **below** the x-axis $\Rightarrow c < 0$.
- Vertex has a positive x-coordinate: $-\dfrac{b}{2a} > 0$. Since $a < 0$, this forces $b > 0$.

Hence $a < 0,\ b > 0,\ c < 0$.

Q4. Correct Option: (b) -7

Since $x = 2$ is a root, substitute into $2x^2 + ax + 6 = 0$:

$$\begin{aligned} 2(2)^2 + a(2) + 6 &= 0 \\ 8 + 2a + 6 &= 0 \\ 2a &= -14 \\ a &= -7 \end{aligned}$$

Q5. Correct Option: (a) $3,\ -5$

For equal roots, $\Delta = 0$:

$$\begin{aligned} [-(m+1)]^2 - 4(1)(4) &= 0 \\ (m+1)^2 &= 16 \\ m + 1 &= \pm 4 \\ m = 3 \quad \text{or} \quad m &= -5 \end{aligned}$$

Q6. Correct Option: (c) $x^2 + x - 56 = 0$
Let the two consecutive natural numbers be x and $x + 1$:

$$x(x+1) = 56$$
$$x^2 + x - 56 = 0$$

Q7. Correct Option: (c) Not real
When $b^2 - 4ac < 0$, the discriminant is negative. The square root of a negative number is not real, so the equation has no real roots.

Q8. Correct Option: (a) $2,\ 0.5$

$$\begin{aligned} x + \frac{1}{x} &= 2.5 \\ x^2 - 2.5x + 1 &= 0 \\ 2x^2 - 5x + 2 &= 0 \quad \text{(multiplying by 2)} \\ (2x-1)(x-2) &= 0 \\ x = \frac{1}{2} &= 0.5 \quad \text{or} \quad x = 2 \end{aligned}$$

Q9. Both A and R are true and R is the correct explanation of A.

Verification of Assertion (A):
For a quadratic equation $ax^2 + bx + c = 0$, if the roots are reciprocals of each other, then their product must be 1. Thus,

$$\alpha\beta = \frac{c}{a} = 1$$

For the given equation $(2k-1)x^2 + 4x - 3 = 0$, we have:

$$\frac{c}{a} = \frac{-3}{2k-1} = 1$$

$$\Rightarrow 2k - 1 = -3$$

$$\Rightarrow 2k = -2 \quad \Rightarrow \quad k = -1$$

Hence, **Assertion (A) is true.**

Verification of Reason (R):
If $a = c$ in a quadratic equation, then:

$$\frac{c}{a} = 1$$

which implies that the product of the roots is 1, and therefore the roots are reciprocals of each other. Hence, **Reason (R) is true.**

Conclusion:
The reason correctly explains the assertion since setting $a = c$ in $(2k-1)x^2 + 4x - 3 = 0$ gives:

$$2k - 1 = -3 \Rightarrow k = -1$$

Therefore, **both A and R are true, and R is the correct explanation of A.**

Q10. Correct Option: (a) 196

For $\sqrt{3}\,x^2 + 10x - 8\sqrt{3} = 0$, here $a = \sqrt{3}$, $b = 10$, $c = -8\sqrt{3}$:

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (10)^2 - 4(\sqrt{3})(-8\sqrt{3}) \\ &= 100 + 4 \times 8 \times 3 \\ &= 100 + 96 = 196 \end{aligned}$$

Q11. Correct Option: (a) $p^2 - p = 2q$

Let the roots of $x^2 - px + q = 0$ be α and β.
By Vieta's formulae: $\alpha + \beta = p$ and $\alpha\beta = q$.
Sum of squares of roots:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 2q$$

Given condition (sum of roots = sum of squares of roots):

$$\begin{aligned} \alpha + \beta &= \alpha^2 + \beta^2 \\ p &= p^2 - 2q \\ p^2 - p &= 2q \end{aligned}$$

Q12. Correct Option: (c) 5

$$\begin{aligned} \sqrt{3x^2 + 6} &= 9 \\ 3x^2 + 6 &= 81 \quad \text{(squaring both sides)} \\ 3x^2 &= 75 \\ x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

The positive root is $x = 5$.

Section B

Q13. [Concept: Nature of Roots]

The equation is $(p - 4)x^2 + 2(p - 4)x + 2 = 0$, $p \neq 4$.

Here $A = p - 4$, $B = 2(p - 4)$, $C = 2$.

For equal roots, the discriminant must be zero:

$$\begin{aligned} \Delta &= 0 \\ [2(p - 4)]^2 - 4(p - 4)(2) &= 0 \\ 4(p - 4)^2 - 8(p - 4) &= 0 \\ 4(p - 4)\left[(p - 4) - 2\right] &= 0 \\ 4(p - 4)(p - 6) &= 0 \\ p = 4 \quad \text{or} \quad p &= 6 \end{aligned}$$

Since $p \neq 4$ (given), the required value is $p = 6$.

Q14. [Concept: Factorisation]

$$\frac{x+1}{x-1}+\frac{x-2}{x+2}=3, \quad x\neq 1,\ -2$$

Multiply throughout by $(x-1)(x+2)$:

$$\begin{aligned}(x+1)(x+2)+(x-2)(x-1)&=3(x-1)(x+2)\\(x^2+3x+2)+(x^2-3x+2)&=3(x^2+x-2)\\2x^2+4&=3x^2+3x-6\\0&=x^2+3x-10\\(x+5)(x-2)&=0\end{aligned}$$

$$x=-5 \quad \text{or} \quad x=2$$

Both values satisfy $x\neq 1,\ -2$, so both are valid solutions.

Q15. [Concept: Quadratic Formula]
For $2x^2-5x-4=0$, here $a=2$, $b=-5$, $c=-4$:

$$\begin{aligned}x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\&=\frac{5\pm\sqrt{(-5)^2-4(2)(-4)}}{2(2)}\\&=\frac{5\pm\sqrt{25+32}}{4}\\&=\frac{5\pm\sqrt{57}}{4}\end{aligned}$$

Since $\sqrt{57}\approx 7.550$:

$$\begin{aligned}x_1&=\frac{5+7.550}{4}=\frac{12.550}{4}\approx 3.1 \qquad &\text{(correct to 2 significant figures)}\\x_2&=\frac{5-7.550}{4}=\frac{-2.550}{4}\approx -0.64 \quad &\text{(correct to 2 significant figures)}\end{aligned}$$

Q16. [Concept: Basic Variable Solving]

$$\sqrt{2x+9}+x=13$$

Isolate the radical:

$$\sqrt{2x+9}=13-x$$

Square both sides (valid only when $13-x\geq 0$, i.e. $x\leq 13$):

$$\begin{aligned}2x+9&=(13-x)^2\\2x+9&=169-26x+x^2\\0&=x^2-28x+160\\(x-8)(x-20)&=0\\x=8 \quad &\text{or} \quad x=20\end{aligned}$$

Check $x = 20$: $\sqrt{49} + 20 = 7 + 20 = 27 \neq 13$ (rejected, extraneous root).
Check $x = 8$: $\sqrt{25} + 8 = 5 + 8 = 13$ ✓

Therefore $x = 8$.

Q17. [Concept: Simple Number Problem]

Let the required number be x. Its reciprocal is $\frac{1}{x}$.

$$\begin{aligned} x + \frac{1}{x} &= \frac{10}{3} \\ 3(x^2 + 1) &= 10x \quad \text{(multiplying by } 3x\text{)} \\ 3x^2 - 10x + 3 &= 0 \\ (3x - 1)(x - 3) &= 0 \\ x = \frac{1}{3} &\quad \text{or} \quad x = 3 \end{aligned}$$

The required numbers are $\frac{1}{3}$ and 3.
(Note: each is the reciprocal of the other, confirming both are valid.)

Section C

Q18. [Word Problem: Distance/Time]

Let the speed of the fast train be v km/h.
Then the speed of the slow train = $(v - 10)$ km/h.

Time taken by fast train = $\frac{600}{v}$ h; Time taken by slow train = $\frac{600}{v-10}$ h.

Given the slow train takes 3 hours more than the fast train:

$$\begin{aligned} \frac{600}{v - 10} - \frac{600}{v} &= 3 \\ 600v - 600(v - 10) &= 3v(v - 10) \quad \text{(multiplying by } v(v - 10)\text{)} \\ 600v - 600v + 6000 &= 3v^2 - 30v \\ 6000 &= 3v^2 - 30v \\ v^2 - 10v - 2000 &= 0 \quad \text{(dividing by 3)} \\ (v - 50)(v + 40) &= 0 \\ v = 50 &\quad \text{or} \quad v = -40 \end{aligned}$$

Since speed cannot be negative, $v = 50$ km/h.

$$\begin{aligned} \text{Speed of fast train} &= 50 \text{ km/h} \\ \text{Speed of slow train} &= 40 \text{ km/h} \end{aligned}$$

Q19. [Case-Based: Rectangular Garden]

(a) Let the original length of the garden = x metres.
Since Area = 192 m^2:

$$\text{Breadth} = \frac{192}{x} \text{ metres}$$

(b) When the length is reduced by 4 m and the breadth is increased by 4 m, the garden becomes a square, so the new length equals the new breadth:

$$\begin{aligned} x - 4 &= \frac{192}{x} + 4 \\ x - 8 &= \frac{192}{x} \\ x(x-8) &= 192 \\ x^2 - 8x - 192 &= 0 \end{aligned}$$

(c) Solving $x^2 - 8x - 192 = 0$ using the quadratic formula:

$$x = \frac{8 \pm \sqrt{64 + 768}}{2} = \frac{8 \pm \sqrt{832}}{2} = \frac{8 \pm 8\sqrt{13}}{2} = 4 \pm 4\sqrt{13}$$

Since $x > 0$, we take $x = 4 + 4\sqrt{13} = 4(1 + \sqrt{13})$ m.
Breadth:

$$\frac{192}{x} = \frac{192}{4(1+\sqrt{13})} = \frac{48}{1+\sqrt{13}} \cdot \frac{\sqrt{13}-1}{\sqrt{13}-1} = \frac{48(\sqrt{13}-1)}{12} = 4(\sqrt{13}-1) \text{ m}$$

Original dimensions:

$$\begin{aligned} \text{Length} &= 4(1+\sqrt{13}) \text{ m} \approx 18.4 \text{ m} \\ \text{Breadth} &= 4(\sqrt{13}-1) \text{ m} \approx 10.4 \text{ m} \end{aligned}$$

Verification: New side $= x - 4 = 4\sqrt{13}$ and new breadth $+4 = 4(\sqrt{13} - 1) + 4 = 4\sqrt{13}$ ✓.

Q20. [Mixed-Concept Question]
For $x^2 - (k+6)x + 2(2k-1) = 0$, by Vieta's formulae:

$$\begin{aligned} \text{Sum of roots} &= k + 6 \\ \text{Product of roots} &= 2(2k-1) = 4k - 2 \end{aligned}$$

Step 1: Find k.
Condition: Sum of roots = half of product of roots:

$$\begin{aligned} k + 6 &= \frac{1}{2}(4k - 2) \\ k + 6 &= 2k - 1 \\ k &= 7 \end{aligned}$$

Step 2: Form the equation with $k = 7$:

$$\begin{aligned} x^2 - (7+6)x + 2(14-1) &= 0 \\ x^2 - 13x + 26 &= 0 \end{aligned}$$

Step 3: Nature of roots.

$$\Delta = (-13)^2 - 4(1)(26) = 169 - 104 = 65$$

Since $\Delta = 65 > 0$ and 65 is not a perfect square, the roots are **real, irrational, and distinct**.

Q21. [Word Problem: Geometry]

Let the shortest side = x cm.

Then: Hypotenuse = $(2x + 2)$ cm (2 more than twice the shortest side).
Third side = $(2x + 2) - 2 = 2x$ cm (2 less than the hypotenuse).

By the Pythagorean theorem (right angle between the two legs):

$$\begin{aligned}(\text{shortest})^2 + (\text{third side})^2 &= (\text{hypotenuse})^2 \\ x^2 + (2x)^2 &= (2x+2)^2 \\ x^2 + 4x^2 &= 4x^2 + 8x + 4 \\ x^2 &= 8x + 4 \\ x^2 - 8x - 4 &= 0\end{aligned}$$

Using the quadratic formula:

$$x = \frac{8 \pm \sqrt{64 + 16}}{2} = \frac{8 \pm \sqrt{80}}{2} = \frac{8 \pm 4\sqrt{5}}{2} = 4 \pm 2\sqrt{5}$$

Since $x > 0$, we take $x = 4 + 2\sqrt{5}$ cm.

Lengths of all sides:

$$\begin{aligned}\text{Shortest side} &= (4 + 2\sqrt{5}) \text{ cm} \\ \text{Third side} = 2x &= (8 + 4\sqrt{5}) \text{ cm} \\ \text{Hypotenuse} = 2x + 2 &= (10 + 4\sqrt{5}) \text{ cm}\end{aligned}$$

Verification:

$$\begin{aligned}(4+2\sqrt{5})^2 + (8+4\sqrt{5})^2 &= (36 + 16\sqrt{5}) + (144 + 64\sqrt{5}) \\ &= 180 + 80\sqrt{5} \\ (10+4\sqrt{5})^2 &= 100 + 80\sqrt{5} + 80 = 180 + 80\sqrt{5} \quad \checkmark\end{aligned}$$

Q22. [High-Difficulty Board Level]

Solution:

Given:

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}, \quad (a, b, x \neq 0)$$

Combining the RHS:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{bx + ax + ab}{abx}$$

So,

$$\frac{1}{a+b+x} = \frac{ax + bx + ab}{abx}$$

Cross-multiplying:

$$abx = (a+b+x)(ax + bx + ab)$$

Now,

$$ax + bx = x(a+b)$$

So,

$$abx = (a+b+x)\big(x(a+b) + ab\big)$$

Let $k = a + b$. Then:

$$abx = (k + x)(kx + ab)$$

Expanding:

$$(k + x)(kx + ab) = k^2x + kx^2 + abk + abx$$

Cancelling abx from both sides:

$$0 = k^2x + kx^2 + abk$$

Factoring:

$$0 = k(kx + x^2 + ab)$$

Since $k \neq 0$,

$$x^2 + kx + ab = 0$$

Substituting back $k = a + b$:

$$x^2 + (a + b)x + ab = 0$$

Factoring:

$$(x + a)(x + b) = 0$$

$$\boxed{x = -a \quad \text{or} \quad x = -b}$$