Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (B) $(-3, -5)$

Reason: Under reflection in the y-axis (line $x = 0$), the x-coordinate changes sign while the y-coordinate remains unchanged.

$$P(3, -5) \xrightarrow{\text{reflect in } y\text{-axis}} (-3, -5)$$

Q2. Correct Option: (B) $(-4, -7)$

Reason: Under reflection in the x-axis (line $y = 0$), the y-coordinate changes sign while the x-coordinate remains unchanged.

$$Q(-4, 7) \xrightarrow{\text{reflect in } x\text{-axis}} (-4, -7)$$

Q3. Correct Option: (A) $(-6, 2)$

Reason: Under reflection in the origin, both coordinates change sign.

$$R(6, -2) \xrightarrow{\text{reflect in origin}} (-6, 2)$$

Q4. Correct Option: (A) $(6, 5)$

Reason: To reflect a point (x, y) in the vertical line $x = a$, the image is $(2a - x,\ y)$. Here $a = 4$, $x = 2$.

$$x' = 2(4) - 2 = 6, \qquad y' = 5$$

$$A(2, 5) \xrightarrow{\text{reflect in } x=4} (6, 5)$$

Q5. Correct Option: (A) $(-3, -5)$

Reason: To reflect a point (x, y) in the horizontal line $y = b$, the image is $(x,\ 2b - y)$. Here $b = -2$, $y = 1$.

$$y' = 2(-2) - 1 = -4 - 1 = -5, \qquad x' = -3$$

$$B(-3, 1) \xrightarrow{\text{reflect in } y=-2} (-3, -5)$$

Q6. Correct Option: (B) Its y-coordinate is 0.

Reason: Reflection in the x-axis maps $(x, y) \to (x, -y)$. For a point to be invariant (map to itself):

$$(x, y) = (x, -y) \implies y = -y \implies 2y = 0 \implies y = 0$$

The point must lie on the x-axis, i.e., its y-coordinate must be 0.

Q7. Correct Option: (C) $(-5,\,-3)$

Reason:

Step 1 — Reflect $P(5,\,3)$ in the y-axis:

$$P(5,\,3) \to P'(-5,\,3)$$

Step 2 — Reflect $P'(-5,\,3)$ in the x-axis:

$$P'(-5,\,3) \to P''(-5,\,-3)$$

Q8. Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Reason:

Checking Assertion A:

Reflect $P(a,\,b)$ in the x-axis first, then in the y-axis:

$$P(a,\,b) \xrightarrow{x\text{-axis}} (a,\,-b) \xrightarrow{y\text{-axis}} (-a,\,-b)$$

Checking via origin directly:

$$P(a,\,b) \xrightarrow{\text{origin}} (-a,\,-b) \quad \checkmark$$

Both routes give $(-a,\,-b)$. **Assertion A is TRUE.**

Reason R correctly states that the reflection in the origin is $(-a,\,-b)$, and also correctly describes the successive-reflection result. **R is TRUE** and directly explains why A holds.

Q9. Correct Option: (B) 5

Reason: The mirror line $x = a$ is the perpendicular bisector of the segment joining $C(1,\,6)$ and $C'(9,\,6)$. Since the y-coordinates are equal, a is simply the midpoint of the x-coordinates:

$$a = \frac{1+9}{2} = \frac{10}{2} = 5$$

Q10. Correct Option: (B) Invariant, since every point on the mirror line maps to itself under reflection in that line.

Reason: By the fundamental property of reflection, the mirror line is the set of all invariant (fixed) points. Any point lying exactly on the line $x = 3$ has zero perpendicular distance from the mirror, so it maps to itself regardless of its y-coordinate.

Q11. Correct Option: (B) $h = 0$ **and** $k = 0$

Reason:

Reflection of $P(h,\,k)$ in the y-axis: $(-h,\,k)$.

Reflection of $P(h,\,k)$ in the x-axis: $(h,\,-k)$.

For both images to coincide:

$$-h = h \implies 2h = 0 \implies h = 0$$
$$k = -k \implies 2k = 0 \implies k = 0$$

Both conditions must hold **simultaneously**, so the only such point is the **origin** (0, 0), i.e., $h = 0$ **and** $k = 0$.

Section B — Short Answer Questions

Q12. *Key formulae used:*

- Reflection in $x = a$: $(x, y) \to (2a - x,\ y)$
- Reflection in $y = b$: $(x, y) \to (x,\ 2b - y)$

Given point: $P(-3, 4)$.

(i) **Reflection in $x = 2$:**

$$\begin{aligned} x' &= 2(2) - (-3) = 4 + 3 = 7 \\ y' &= 4 \end{aligned}$$

$$P(-3, 4) \xrightarrow{x=2} (\mathbf{7},\ \mathbf{4})$$

(ii) **Reflection in $y = 1$:**

$$\begin{aligned} x' &= -3 \\ y' &= 2(1) - 4 = 2 - 4 = -2 \end{aligned}$$

$$P(-3, 4) \xrightarrow{y=1} (\mathbf{-3},\ \mathbf{-2})$$

(iii) **Reflection in $x = -1$:**

$$\begin{aligned} x' &= 2(-1) - (-3) = -2 + 3 = 1 \\ y' &= 4 \end{aligned}$$

$$P(-3, 4) \xrightarrow{x=-1} (\mathbf{1},\ \mathbf{4})$$

Q13. *Key formula:* Reflection in the origin maps $(x, y) \to (-x, -y)$.

Given: $A'(5, -3)$ is the image of A under reflection in the origin.

(i) **Finding co-ordinates of A:**
Since $A \to A'$ under reflection in the origin:

$$A' = (-x_A,\ -y_A) = (5,\ -3)$$

$$-x_A = 5 \implies x_A = -5$$
$$-y_A = -3 \implies y_A = 3$$

$$\therefore \quad A = (\mathbf{-5},\ \mathbf{3})$$

(ii) **Verification — origin is midpoint of AA':**

$$\text{Midpoint of } AA' = \left(\frac{-5+5}{2},\ \frac{3+(-3)}{2}\right) = \left(\frac{0}{2},\ \frac{0}{2}\right) = (0,\ 0) \quad \checkmark$$

The origin is indeed the midpoint of AA'.

(iii) **Distance AA':**

$$\begin{aligned} AA' &= \sqrt{(5-(-5))^2 + (-3-3)^2} \\ &= \sqrt{(10)^2 + (-6)^2} \\ &= \sqrt{100+36} \\ &= \sqrt{136} = 2\sqrt{34} \approx \mathbf{11.66} \text{ units} \end{aligned}$$

Q14. *A point is invariant under a reflection if and only if it lies on the mirror line (or, in the case of the origin, at the origin itself).*

(i) **Reflection in $y = 3$:**
Image of $(x,\ y)$ is $(x,\ 6-y)$. For invariance:

$$y = 6 - y \implies 2y = 6 \implies y = 3$$

Invariant set: All points of the form $(x,\ 3)$ for any x, i.e., **every point on the line $y = 3$.**

(ii) **Reflection in $x = -2$:**
Image of $(x,\ y)$ is $(-4-x,\ y)$. For invariance:

$$x = -4 - x \implies 2x = -4 \implies x = -2$$

Invariant set: All points of the form $(-2,\ y)$ for any y, i.e., **every point on the line $x = -2$.**

(iii) **Reflection in the origin:**
Image of $(x,\ y)$ is $(-x,\ -y)$. For invariance:

$$\begin{aligned} x = -x &\implies 2x = 0 \implies x = 0 \\ y = -y &\implies 2y = 0 \implies y = 0 \end{aligned}$$

Both conditions must hold simultaneously. *Invariant set:* **Only the origin** $(0,\ 0)$ — a single point, not a line.

Q15. Vertices: $A(2,\ 3)$, $B(-1,\ 4)$, $C(3,\ -2)$.

(i) **Reflection in the x-axis** $(x,\ y) \to (x,\ -y)$:

$$\begin{aligned} A(2,\ 3) &\to A'(2,\ -3) \\ B(-1,\ 4) &\to B'(-1,\ -4) \\ C(3,\ -2) &\to C'(3,\ 2) \end{aligned}$$

(ii) **Reflection of A', B', C' in the y-axis** $(x,\, y) \to (-x,\, y)$:

$$\begin{aligned} A'(2,\, -3) &\to A''(-2,\, -3) \\ B'(-1,\, -4) &\to B''(1,\, -4) \\ C'(3,\, 2) &\to C''(-3,\, 2) \end{aligned}$$

(iii) **Single equivalent transformation:**
Comparing $A(2,\, 3) \to A''(-2,\, -3)$, $B(-1,\, 4) \to B''(1,\, -4)$, $C(3,\, -2) \to C''(-3,\, 2)$: in each case both coordinates change sign. This is precisely **reflection in the origin**, which maps $(x,\, y) \to (-x,\, -y)$.

Q16. Given: $P(m,\, 7)$ is reflected in $x = 4$ to give $P'(10,\, n)$.
Formula for reflection in $x = a$: $x' = 2a - x$, $y' = y$.

(i) **Finding m:**

$$\begin{aligned} x' &= 2(4) - m \\ 10 &= 8 - m \\ m &= 8 - 10 = \mathbf{-2} \end{aligned}$$

(ii) **Finding n:**
Reflection in a vertical line leaves the y-coordinate unchanged:

$$n = 7$$

$$\therefore \quad n = \mathbf{7}$$

(iii) **Co-ordinates of P and P':**

$$P = (\mathbf{-2},\, \mathbf{7}), \qquad P' = (\mathbf{10},\, \mathbf{7})$$

Verification: Midpoint of $PP' = \left(\dfrac{-2+10}{2},\, \dfrac{7+7}{2}\right) = (4,\, 7)$, which lies on the line $x = 4$. ✓

Section C — Long Answer Questions

Q17. *Formula used:* Reflection in $y = b$: $(x,\, y) \to (x,\ 2b - y)$. Here $b = 2$, so $(x,\, y) \to (x,\ 4 - y)$.

(i) **Images of the corners under reflection in $y = 2$:**

$$\begin{aligned} A(1,\, 5) &\to A'(1,\ 4-5) = A'(1,\, -1) \\ B(5,\, 5) &\to B'(5,\ 4-5) = B'(5,\, -1) \\ C(5,\, 7) &\to C'(5,\ 4-7) = C'(5,\, -3) \\ D(1,\, 7) &\to D'(1,\ 4-7) = D'(1,\, -3) \end{aligned}$$

(ii) **Co-ordinates of reflected rectangle $A'B'C'D'$:**

$$A'(1,\, -1), \quad B'(5,\, -1), \quad C'(5,\, -3), \quad D'(1,\, -3)$$

(iii) **Total area of both rectangles combined:**
Dimensions of the original rectangle $ABCD$:

$$\text{Width} = 5 - 1 = 4 \text{ m}$$
$$\text{Height} = 7 - 5 = 2 \text{ m}$$
$$\text{Area of } ABCD = 4 \times 2 = 8 \text{ m}^2$$

Reflection preserves shape and size, so the reflected rectangle $A'B'C'D'$ has the same area $= 8$ m^2. Since there is no overlap:

$$\text{Total area} = 8 + 8 = \mathbf{16} \text{ m}^{\mathbf{2}}$$

(iv) **Invariant vertices:**
A point is invariant under reflection in $y = 2$ if and only if its y-coordinate equals 2. Checking the vertices:

$$A:\ y = 5, \quad B:\ y = 5, \quad C:\ y = 7, \quad D:\ y = 7$$

No vertex has $y = 2$. Therefore **no vertex of the original rectangle is invariant** under this reflection.

Q18. *Formula:* Reflection in the origin: $(x, y) \to (-x, -y)$.

(i) **New positions after reflection in the origin:**

$$L(3, 4) \to L'(-3, -4)$$
$$K(-2, 5) \to K'(2, -5)$$
$$S(0, -3) \to S'(0, 3)$$

(ii) **Distance $d_1 = LK$:**

$$d_1 = \sqrt{(3-(-2))^2 + (4-5)^2}$$
$$= \sqrt{(5)^2 + (-1)^2}$$
$$= \sqrt{25+1} = \sqrt{26} \text{ units}$$

(iii) **Distance $d_2 = L'K'$:**

$$d_2 = \sqrt{26} \text{ units}$$

Reason: Reflection is an isometry (distance-preserving transformation). The distance between any two points equals the distance between their images. Hence $d_2 = d_1 = \sqrt{26}$, without further calculation.

(iv) **Invariance of School $S(0, -3)$:**
Image of $S(0, -3)$ under reflection in the origin:

$$S' = (0, 3) \neq (0, -3) = S$$

Therefore S is **not invariant**.

A point (x, y) is invariant under reflection in the origin only if $(x, y) = (-x, -y)$, which gives $x = 0$ and $y = 0$. The **only invariant point** is the **origin** $(0, 0)$.

Q19. Quadrilateral $PQRS$: $P(1,\,3)$, $Q(4,\,3)$, $R(4,\,-1)$, $S(1,\,-1)$.

(i) **Step 1 — Reflect in the y-axis:** $(x,\,y) \to (-x,\,y)$

$$\begin{aligned} P(1,\,3) &\to P'(-1,\,3)\\ Q(4,\,3) &\to Q'(-4,\,3)\\ R(4,\,-1) &\to R'(-4,\,-1)\\ S(1,\,-1) &\to S'(-1,\,-1)\end{aligned}$$

(ii) **Step 2 — Reflect in $y=-1$:** $(x,\,y) \to (x,\,-2-y)$

$$\begin{aligned} P'(-1,\,3) &\to P''(-1,\,-2-3) = P''(-1,\,-5)\\ Q'(-4,\,3) &\to Q''(-4,\,-2-3) = Q''(-4,\,-5)\\ R'(-4,\,-1) &\to R''(-4,\,-2-(-1)) = R''(-4,\,-1)\\ S'(-1,\,-1) &\to S''(-1,\,-2-(-1)) = S''(-1,\,-1)\end{aligned}$$

(iii) **Step 3 — Reflect in the origin:** $(x,\,y) \to (-x,\,-y)$

$$\begin{aligned} P''(-1,\,-5) &\to P'''(1,\,5)\\ Q''(-4,\,-5) &\to Q'''(4,\,5)\\ R''(-4,\,-1) &\to R'''(4,\,1)\\ S''(-1,\,-1) &\to S'''(1,\,1)\end{aligned}$$

(iv) **Invariant vertices of $PQRS$ under the reflection in Step 2 (reflection in $y=-1$):**
A point is invariant under reflection in $y=-1$ if and only if its y-coordinate equals -1. Checking the vertices of the *original* quadrilateral $PQRS$:

$$\begin{aligned} P(1,\,3):&\quad y = 3 \neq -1 \quad \times\\ Q(4,\,3):&\quad y = 3 \neq -1 \quad \times\\ R(4,\,-1):&\quad y = -1 \quad \checkmark\\ S(1,\,-1):&\quad y = -1 \quad \checkmark\end{aligned}$$

Vertices R and S of the original quadrilateral are invariant under the reflection in Step 2, since both lie on the line $y=-1$.

Q20. Let $P = (a,\,b)$.

(i) **Co-ordinates of P_1 and P_2:**

$$P_1 = \text{reflection of } P \text{ in } x\text{-axis} = (a,\,-b)$$

$$P_2 = \text{reflection of } P \text{ in } y\text{-axis} = (-a,\,b)$$

(ii) **Substituting into the line equations:**
$P_1(a,\,-b)$ lies on $y = x - 7$:

$$\begin{aligned} -b &= a - 7\\ a + b &= 7 \quad \cdots (1)\end{aligned}$$

$P_2(-a,\, b)$ lies on $y = 2x + 10$:

$$\begin{aligned} b &= 2(-a) + 10 \\ 2a + b &= 10 \quad \cdots (2) \end{aligned}$$

(iii) **Solving the system:**
Subtracting equation (1) from equation (2):

$$\begin{aligned} (2a + b) - (a + b) &= 10 - 7 \\ a &= 3 \end{aligned}$$

Substituting $a = 3$ into equation (1):

$$\begin{aligned} 3 + b &= 7 \\ b &= 4 \end{aligned}$$

(iv) **Explicit co-ordinates:**

$$P = (a,\, b) = (\mathbf{3},\, \mathbf{4})$$

$$P_1 = (a,\, -b) = (\mathbf{3},\, -\mathbf{4})$$

$$P_2 = (-a,\, b) = (-\mathbf{3},\, \mathbf{4})$$

Verification: $P_1(3,\, -4)$: $x - 7 = 3 - 7 = -4 = y$ ✓ $P_2(-3,\, 4)$: $2(-3) + 10 = 4 = y$ ✓

Q21. Vertices: $A(-2,\, 3)$, $B(4,\, 1)$, $C(1,\, -3)$.

(i) **Reflection in $y = x$:** $(p,\, q) \to (q,\, p)$

$$\begin{aligned} A(-2,\, 3) &\to A'(3,\, -2) \\ B(4,\, 1) &\to B'(1,\, 4) \\ C(1,\, -3) &\to C'(-3,\, 1) \end{aligned}$$

(ii) **Reflection of A', B', C' in the x-axis:** $(x,\, y) \to (x,\, -y)$

$$\begin{aligned} A'(3,\, -2) &\to A''(3,\, 2) \\ B'(1,\, 4) &\to B''(1,\, -4) \\ C'(-3,\, 1) &\to C''(-3,\, -1) \end{aligned}$$

(iii) **Finding original co-ordinates of D given $D''(5,\, -2)$:**
We reverse each step in order.
Reverse Step 2 (reverse reflection in x-axis, which is its own inverse):

$$D''(5,\, -2) \xrightarrow{\text{undo } x\text{-axis}} D'(5,\, 2)$$

Reverse Step 1 (reverse reflection in $y = x$, which is also its own inverse):

$$D'(5,\, 2) \xrightarrow{\text{undo } y=x} D(2,\, 5)$$

$$\therefore \quad \text{Original co-ordinates of } D = (\mathbf{2},\, \mathbf{5})$$

Verification: $D(2,\, 5) \xrightarrow{y=x} (5,\, 2) \xrightarrow{x\text{-axis}} (5,\, -2) = D''$ ✓

(iv) **Point on segment BC invariant under reflection in the x-axis:**

A point is invariant under reflection in the x-axis if and only if $y = 0$. We find where segment BC crosses the x-axis.

Equation of line BC, with $B(4,\,1)$ and $C(1,\,-3)$:

$$\text{Slope} = \frac{-3-1}{1-4} = \frac{-4}{-3} = \frac{4}{3}$$

$$\begin{aligned} y - 1 &= \frac{4}{3}(x-4) \\ y &= \frac{4}{3}x - \frac{16}{3} + 1 = \frac{4}{3}x - \frac{13}{3} \end{aligned}$$

Setting $y = 0$:

$$\begin{aligned} 0 &= \frac{4}{3}x - \frac{13}{3} \\ \frac{4}{3}x &= \frac{13}{3} \\ x &= \frac{13}{4} \end{aligned}$$

$$\therefore \quad \text{The invariant point on } BC = \left(\frac{13}{4},\ 0\right)$$