
Solutions

Section A — Multiple Choice Questions

Q1. **Correct Option: (A)** $3 \times 21 = 7 \times 9$

Reason: In a proportion $a : b = c : d$, the rule is:

$$\text{Product of Extremes} = \text{Product of Means}$$

For 3, 7, 9, 21, the extremes are 3 and 21; the means are 7 and 9.

$$\text{Product of Extremes} = 3 \times 21 = 63$$

$$\text{Product of Means} = 7 \times 9 = 63$$

Since both products are equal, option (A) correctly verifies the proportion.

Q2. **Correct Option: (C)** 10

Reason: If m is the mean proportion between p and q , then:

$$m = \sqrt{p \times q}$$

$$m = \sqrt{4 \times 25}$$

$$= \sqrt{100}$$

$$= 10$$

Q3. **Correct Option: (A)** $a : b = b : c$

Reason: By definition, three quantities a , b , c are said to be in *continued proportion* when:

$$a : b = b : c \implies b^2 = ac$$

Option (A) states exactly this definition and is therefore correct.

Note: Option (C) states $b^2 = a + c$ and option (D) states $a \cdot c = b + b$, both of which are incorrect.

Q4. **Correct Option: (B)** $\frac{8}{5}$

Reason: The **Componendo** property states:

$$\text{If } \frac{x}{y} = \frac{p}{q}, \text{ then } \frac{x+y}{y} = \frac{p+q}{q}$$

Given $\frac{x}{y} = \frac{3}{5}$:

$$\begin{aligned} \frac{x+y}{y} &= \frac{3+5}{5} \\ &= \frac{8}{5} \end{aligned}$$

Q5. Correct Option: (A) $\frac{3}{4}$

Reason: The **Dividendo** property states:

$$\text{If } \frac{p}{q} = \frac{m}{n}, \text{ then } \frac{p-q}{q} = \frac{m-n}{n}$$

Given $\frac{p}{q} = \frac{7}{4}$:

$$\begin{aligned} \frac{p-q}{q} &= \frac{7-4}{4} \\ &= \frac{3}{4} \end{aligned}$$

Q6. Correct Option: (B) $a : c = b : d$

Reason: The **Alternendo** property states:

$$\text{If } a : b = c : d, \text{ then } a : c = b : d$$

(The means b and c are interchanged.) This matches option **(B)**.

Note: Option (A) is the *Invertendo* property; option (D) is the *Componendo* property.

Q7. [Assertion–Reason]

Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Verification of the Assertion:

Starting from $\frac{a}{b} = \frac{c}{d}$, apply the **Componendo–Dividendo** property in two steps:

Step 1 — Componendo:

$$\frac{a+b}{b} = \frac{c+d}{d} \quad \dots (1)$$

Step 2 — Dividendo:

$$\frac{a-b}{b} = \frac{c-d}{d} \quad \dots (2)$$

Dividing (1) by (2):

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

The Assertion is **true**. The Reason correctly attributes this result to the simultaneous application of componendo and dividendo, so **R is the correct explanation of A**.

Q8. Correct Option: (B) 36

Reason: If x , 6, y are in continued proportion:

$$\frac{x}{6} = \frac{6}{y}$$

$$\Rightarrow xy = 6 \times 6 = \mathbf{36}$$

The value of xy is determined entirely by the continued proportion condition. (The additional condition $x + y = 25$ confirms the existence of a real solution but does not change the product.)

Q9. Correct Option: (D) All of them

Reason: Given $\frac{a}{b} = \frac{c}{d}$, verify each option:

Option	Statement	Property Used	Valid?
(A)	$\frac{b}{a} = \frac{d}{c}$	Invertendo	✓
(B)	$\frac{a}{c} = \frac{b}{d}$	Alternendo	✓
(C)	$\frac{a+c}{b+d} = \frac{a}{b}$	Equal Ratios / Addendo	✓

All three statements are valid consequences of the given proportion, so **(D) All of them** is correct.

Q10. [Non-Routine]

Correct Option: (B) 6 : 8 : 9

Reason: Given $a : b = 3 : 4$ and $b : c = 8 : 9$.

To combine these ratios, make b common by finding $\text{LCM}(4, 8) = 8$.

$$a : b = 3 : 4 = (3 \times 2) : (4 \times 2) = 6 : 8$$

$$b : c = 8 : 9 \quad (\text{unchanged})$$

Now $b = 8$ is the common link:

$$a : b : c = 6 : 8 : 9$$

Q11. [Non-Routine / Tricky]

Correct Option: (A) a, b, c, d are in continued proportion

Reason:

Condition 1: b is the mean proportion between a and c :

$$b^2 = ac \quad \Rightarrow \quad \frac{a}{b} = \frac{b}{c} \quad \dots (1)$$

Condition 2: c is the mean proportion between b and d :

$$c^2 = bd \quad \Rightarrow \quad \frac{b}{c} = \frac{c}{d} \quad \dots (2)$$

Combining (1) and (2):

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

This is precisely the definition of a, b, c, d being in **continued proportion**.

Check on other options:

Option (B): $a^2 = cd$ — not generally true.

Option (C): $b^2 = ad$ — not generally true (we have $b^2 = ac$).

Option (D): $a : d = b : c$ — not a necessary consequence.

Solutions

Section B — Short Answer Questions

Q12. Find the value of x if x is the mean proportion between $(x - 3)$ and $(x + 5)$.

Concept: If x is the mean proportion between p and q , then $x^2 = p \cdot q$.

Setting $p = x - 3$ and $q = x + 5$:

$$x^2 = (x - 3)(x + 5)$$

$$x^2 = x^2 + 5x - 3x - 15$$

$$x^2 = x^2 + 2x - 15$$

$$0 = 2x - 15$$

$$2x = 15$$

$$x = \frac{15}{2}$$

Verification:

$$p = x - 3 = \frac{15}{2} - 3 = \frac{9}{2}, \quad q = x + 5 = \frac{15}{2} + 5 = \frac{25}{2}$$

$$p \times q = \frac{9}{2} \times \frac{25}{2} = \frac{225}{4} = \left(\frac{15}{2}\right)^2 = x^2 \quad \checkmark$$

$$x = \frac{15}{2} = 7.5$$

Q13. If a, b, c are in continued proportion and $a = 4, b = 6$, find:

Concept: a, b, c in continued proportion $\Rightarrow \frac{a}{b} = \frac{b}{c}$, i.e. $b^2 = ac$.

(i) **The value of c :**

$$b^2 = ac$$

$$(6)^2 = 4 \times c$$

$$36 = 4c$$

$$c = \frac{36}{4} = 9$$

$$c = 9$$

(ii) **The value of $\frac{a^2 + b^2}{b^2 + c^2}$:**

Using $a = 4, b = 6, c = 9$:

$$a^2 + b^2 = (4)^2 + (6)^2 = 16 + 36 = 52$$

$$b^2 + c^2 = (6)^2 + (9)^2 = 36 + 81 = 117$$

$$\frac{a^2 + b^2}{b^2 + c^2} = \frac{52}{117}$$

Simplifying by dividing both terms by their HCF = 13:

$$\frac{52}{117} = \frac{52 \div 13}{117 \div 13} = \frac{4}{9}$$

Cross-check using proportion: Since $\frac{a}{b} = \frac{4}{6} = \frac{b}{c} = \frac{6}{9} = \frac{2}{3}$, the ratio $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c} = \frac{4}{9}$ is consistent with this being a geometric proportion. ✓

$$\frac{a^2 + b^2}{b^2 + c^2} = \frac{4}{9}$$

Q14. Using componendo and dividendo, find the value of x if:

$$\frac{x+3}{x-3} = \frac{5}{2}$$

Concept — Componendo and Dividendo:

$$\text{If } \frac{p}{q} = \frac{r}{s}, \quad \text{then } \frac{p+q}{p-q} = \frac{r+s}{r-s}$$

The given ratio is already in the form $\frac{(x+3)}{(x-3)} = \frac{5}{2}$, which may be read as $p = x+3$, $q = x-3$, $r = 5$, $s = 2$.

Observe that $p+q = (x+3) + (x-3) = 2x$ and $p-q = (x+3) - (x-3) = 6$.

Applying componendo and dividendo to $\frac{p}{q} = \frac{r}{s}$ gives $\frac{p+q}{p-q} = \frac{r+s}{r-s}$:

$$\frac{(x+3) + (x-3)}{(x+3) - (x-3)} = \frac{5+2}{5-2}$$

$$\frac{2x}{6} = \frac{7}{3}$$

$$\frac{x}{3} = \frac{7}{3}$$

$$x = 7$$

Verification:

$$\frac{7+3}{7-3} = \frac{10}{4} = \frac{5}{2} \quad \checkmark$$

$$x = 7$$

Q15. Given $\frac{2a+3b}{2a-3b} = \frac{5}{3}$, use componendo and dividendo to find $a : b$.

Concept — Componendo and Dividendo:

$$\frac{p}{q} = \frac{r}{s} \implies \frac{p+q}{p-q} = \frac{r+s}{r-s}$$

Here $p = 2a + 3b$, $q = 2a - 3b$, $r = 5$, $s = 3$.

Applying componendo and dividendo:

$$\frac{(2a + 3b) + (2a - 3b)}{(2a + 3b) - (2a - 3b)} = \frac{5 + 3}{5 - 3}$$

$$\frac{4a}{6b} = \frac{8}{2}$$

$$\frac{4a}{6b} = 4$$

$$\frac{a}{b} = 4 \times \frac{6}{4}$$

$$\frac{a}{b} = 6$$

Therefore $a : b = 6 : 1$.

Verification:

$$\frac{2(6) + 3(1)}{2(6) - 3(1)} = \frac{12 + 3}{12 - 3} = \frac{15}{9} = \frac{5}{3} \quad \checkmark$$

$$a : b = 6 : 1$$

Q16. Find the fourth proportional to 5, 8, and 15. Also verify using the fundamental property of proportion.

Concept: If a , b , c , d are in proportion then $a : b = c : d$, which gives $ad = bc$.

Let the fourth proportional be d . Then:

$$5 : 8 = 15 : d$$

$$\frac{5}{8} = \frac{15}{d}$$

$$5d = 8 \times 15$$

$$5d = 120$$

$$d = \frac{120}{5}$$

$$d = 24$$

Verification using fundamental property of proportion (Product of Extremes = Product of Means):

$$\text{Product of Extremes} = 5 \times 24 = 120$$

$$\text{Product of Means} = 8 \times 15 = 120$$

$$120 = 120 \quad \checkmark$$

The fourth proportional to 5, 8, 15 is 24.

Solutions

Section C — Long Answer Questions

Q17. [Word Problem] Notebooks and Pens

Given: Ratio of notebooks to pens = 7 : 3; total items = 1200; cost/notebook = Rs 12; cost/pen = Rs 5.

(i) **Number of notebooks and pens:**

Let number of notebooks = $7x$ and pens = $3x$.

$$7x + 3x = 1200$$

$$10x = 1200$$

$$x = 120$$

$$\text{Notebooks} = 7 \times 120 = 840$$

$$\text{Pens} = 3 \times 120 = 360$$

Notebooks = 840, Pens = 360

(ii) **Total cost of notebooks and pens separately:**

$$\text{Cost of notebooks} = 840 \times 12 = \text{Rs } 10,080$$

$$\text{Cost of pens} = 360 \times 5 = \text{Rs } 1,800$$

Cost of notebooks = Rs 10,080; Cost of pens = Rs 1,800

(iii) **Ratio of total cost of notebooks to total cost of pens:**

$$\text{Ratio} = 10080 : 1800$$

$$= \frac{10080}{1800} = \frac{10080 \div 360}{1800 \div 360} = \frac{28}{5}$$

Ratio = 28 : 5

Q18. [Word Problem] Similar Triangles

Given: Ratio of corresponding sides of two similar triangles = 3 : 5.

(i) Perimeter of the larger triangle:

For similar triangles, ratio of perimeters = ratio of corresponding sides.

$$\frac{48}{P_{\text{large}}} = \frac{3}{5}$$

$$P_{\text{large}} = \frac{48 \times 5}{3} = \mathbf{80 \text{ cm}}$$

Perimeter of larger triangle = 80 cm

(ii) Area of the larger triangle:

Ratio of areas = (ratio of sides)²:

$$\frac{A_{\text{small}}}{A_{\text{large}}} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$\frac{108}{A_{\text{large}}} = \frac{9}{25}$$

$$A_{\text{large}} = \frac{108 \times 25}{9} = \mathbf{300 \text{ cm}^2}$$

Area of larger triangle = 300 cm²

(iii) Corresponding height of the larger triangle:

Ratio of heights = ratio of corresponding sides.

$$\frac{12}{h_{\text{large}}} = \frac{3}{5}$$

$$h_{\text{large}} = \frac{12 \times 5}{3} = \mathbf{20 \text{ cm}}$$

Height of larger triangle = 20 cm

Q19. [Case-Based] Water Distribution System

Given: $P = 200 \text{ L}$, $Q = 400 \text{ L}$; P, Q, R in continued proportion.

(i) Volume of water in tank R :

Continued proportion $\Rightarrow Q^2 = P \times R$.

$$(400)^2 = 200 \times R$$

$$160000 = 200R$$

$$R = \mathbf{800 \text{ litres}}$$

Volume in tank $R = 800 \text{ litres}$

(ii) **Verification of continued proportion:**

$$\frac{P}{Q} = \frac{200}{400} = \frac{1}{2}, \quad \frac{Q}{R} = \frac{400}{800} = \frac{1}{2}$$

Since $\frac{P}{Q} = \frac{Q}{R}$ and $Q^2 = 160000 = P \times R$, P, Q, R are confirmed to be in continued proportion. ✓

(iii) **Finding k such that new $P : Q = 5 : 9$:**

After adding k litres to each:

$$\frac{200 + k}{400 + k} = \frac{5}{9}$$

$$9(200 + k) = 5(400 + k)$$

$$1800 + 9k = 2000 + 5k$$

$$4k = 200$$

$$k = \mathbf{50 \text{ litres}}$$

$k = 50$ litres

(iv) **Mean proportion between volumes of tanks P and R :**

$$\text{Mean proportion} = \sqrt{P \times R} = \sqrt{200 \times 800} = \sqrt{160000} = \mathbf{400 \text{ litres}}$$

This equals Q , confirming Q is the geometric mean in the continued proportion. ✓

Mean proportion = 400 litres

Q20. Let the number of textbooks be $8x$ and the number of reference journals be $3x$.
Given the total number of books is 440:

$$8x + 3x = 440$$

$$11x = 440$$

$$x = 40$$

$$\text{Number of textbooks} = 8 \times 40 = 320$$

$$\text{Number of reference journals} = 3 \times 40 = 120$$

Answer (i): 320 textbooks and 120 reference journals.

First, calculate the total cost before the discount:

$$\text{Total cost of textbooks} = 320 \times 250 = \text{Rs } 80,000$$

$$\text{Total cost of reference journals} = 120 \times 600 = \text{Rs } 72,000$$

Now, calculate the discounts:

$$\text{Discount on textbooks} = 10\% \text{ of } 80,000 = \frac{10}{100} \times 80,000 = \text{Rs } 8,000$$

$$\text{Discounted price of textbooks} = 80,000 - 8,000 = \text{Rs } 72,000$$

$$\text{Discount on reference journals} = 5\% \text{ of } 72,000 = \frac{5}{100} \times 72,000 = \text{Rs } 3,600$$

$$\text{Discounted price of reference journals} = 72,000 - 3,600 = \text{Rs } 68,400$$

Final total amount paid = $72,000 + 68,400 = \text{Rs } 1,40,400$

Answer (ii): The final amount paid is Rs 1,40,400.

Q21. Total profit earned = Rs 81,000.

Amount reinvested = 20% of the total profit.

$$\begin{aligned}\text{Reinvested Amount} &= \frac{20}{100} \times 81,000 \\ &= 2 \times 8,100 \\ &= \text{Rs } 16,200\end{aligned}$$

Answer (i): The amount reinvested is Rs 16,200.

First, find the remaining profit to be shared:

Remaining profit = Total Profit – Reinvested Amount

Remaining profit = $81,000 - 16,200 = \text{Rs } 64,800$

This remaining profit is divided in the ratio of their investments, which is 4 : 5.

Sum of the ratios = $4 + 5 = 9$.

Aman's share of the profit:

$$\begin{aligned}\text{Aman's share} &= \frac{4}{9} \times 64,800 \\ &= 4 \times 7,200 \\ &= \text{Rs } 28,800\end{aligned}$$

Raj's share of the profit:

$$\begin{aligned}\text{Raj's share} &= \frac{5}{9} \times 64,800 \\ &= 5 \times 7,200 \\ &= \text{Rs } 36,000\end{aligned}$$

Answer (ii): Aman receives Rs 28,800 and Raj receives Rs 36,000.