Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (B) (2, 2)

Reason: Using the midpoint formula:

$$M = \left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right) = \left(\frac{6 + (-2)}{2},\ \frac{-4 + 8}{2}\right) = \left(\frac{4}{2},\ \frac{4}{2}\right) = (2,\ 2)$$

Q2. Correct Option: (C) (5, 7)

Reason: Using the internal section formula with $m : n = 2 : 1$, $P(1,\ 3)$, $Q(7,\ 9)$:

$$x = \frac{m \cdot x_2 + n \cdot x_1}{m + n} = \frac{2(7) + 1(1)}{3} = \frac{15}{3} = 5$$

$$y = \frac{m \cdot y_2 + n \cdot y_1}{m + n} = \frac{2(9) + 1(3)}{3} = \frac{21}{3} = 7$$

Q3. Correct Option: (C) (4, 4)

Reason: The centroid G of a triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) is:

$$G = \left(\frac{x_1 + x_2 + x_3}{3},\ \frac{y_1 + y_2 + y_3}{3}\right) = \left(\frac{2 + 6 + 4}{3},\ \frac{4 + 0 + 8}{3}\right) = \left(\frac{12}{3},\ \frac{12}{3}\right) = (4,\ 4)$$

Q4. Correct Option: (B) $2 : 1$

Reason: Let $P(3,\ 5)$ divide $A(-1,\ 1)$ and $B(5,\ 7)$ in ratio $m : n$. Using the x-coordinate:

$$\begin{aligned} 3 &= \frac{m(5) + n(-1)}{m + n} \\ 3(m + n) &= 5m - n \\ 3m + 3n &= 5m - n \\ 4n &= 2m \\ \frac{m}{n} &= \frac{4}{2} = \frac{2}{1} \end{aligned}$$

Verification using y-coordinate: $\dfrac{2(7) + 1(1)}{3} = \dfrac{15}{3} = 5$ ✓

Q5. Correct Option: (A) $x = 7,\ y = 10$

Reason: Using the midpoint formula:

$$\frac{x + 3}{2} = 5 \implies x + 3 = 10 \implies x = 7$$

$$\frac{4 + y}{2} = 7 \implies 4 + y = 14 \implies y = 10$$

Q6. Correct Option: (A) $1 : 1$

Reason: Since P lies on the x-axis, its y-coordinate is 0. Let P divide $A(3, 6)$ and $B(9, -6)$ internally in ratio $m : n$.

Using the y-coordinate from the section formula:

$$\begin{aligned}\frac{m(-6) + n(6)}{m + n} &= 0 \\ -6m + 6n &= 0 \\ 6n &= 6m \\ \frac{m}{n} &= \frac{1}{1}\end{aligned}$$

Hence the ratio is $\mathbf{1} : \mathbf{1}$.

Finding the co-ordinates of P:

$$x_P = \frac{1(9) + 1(3)}{1 + 1} = \frac{12}{2} = 6$$

$$y_P = \frac{1(-6) + 1(6)}{2} = \frac{0}{2} = 0 \quad \checkmark$$

Thus $P = (6, 0)$, which correctly lies on the x-axis.

Q7. Correct Option: (B) Both A and R are true, but R is not the correct explanation of A.

Reason:

- **Assertion A is TRUE.** The centroid is the intersection of the medians and always lies inside the triangle, for any triangle (acute, obtuse, or right-angled).
- **Reason R is TRUE.** The centroid divides each median in ratio $2 : 1$ from the vertex, and the coordinate formula stated is correct.
- However, **R does not explain A.** The formula for the centroid's coordinates tells us *where* the centroid is, not *why* it must lie inside the triangle. The fact that it lies inside follows from the geometric property of medians, not from the coordinate formula alone. Hence R is not the correct explanation of A.

Q8. Correct Option: (B) 2

Reason: P divides $A(1, -1)$ and $B(9, 7)$ in ratio $1 : 3$.

Using the section formula for x-coordinate:

$$x_P = \frac{1(9) + 3(1)}{1 + 3} = \frac{9 + 3}{4} = \frac{12}{4} = 3$$

But $x_P = k + 1$, so:

$$k + 1 = 3 \implies k = 2$$

Verification using y-coordinate:

$$y_P = \frac{1(7) + 3(-1)}{4} = \frac{4}{4} = 1$$

$2k - 1 = 2(2) - 1 = 3 \neq 1.$

Re-checking: $y_P = 1$ and $2k - 1 = 1 \Rightarrow k = 1$. Using x: $k + 1 = 3 \Rightarrow k = 2$.

Since both coordinates must be consistent, let us use both:

$$\begin{aligned} k + 1 = 3 &\implies k = 2 \quad \text{(from } x\text{)} \\ 2k - 1 = 1 &\implies k = 1 \quad \text{(from } y\text{)} \end{aligned}$$

There is a contradiction, meaning $P(k + 1, 2k - 1)$ with a single k cannot satisfy both simultaneously for ratio $1 : 3$. The question uses the x-coordinate to determine k:

$$k + 1 = 3 \implies \boxed{k = 2}$$

The intended answer based on the x-coordinate condition is $\mathbf{k = 2}$.

Q9. Correct Option: (A) $(4,\ 5)$

Reason: Let the third vertex be $C(x,\ y)$. Using the centroid formula with $G(2,\ 3)$:

$$\frac{4 + (-2) + x}{3} = 2 \implies 2 + x = 6 \implies x = 4$$
$$\frac{-2 + 6 + y}{3} = 3 \implies 4 + y = 9 \implies y = 5$$

$$C = (4,\ 5)$$

Q10. Correct Option: (A) $\left(5,\ \frac{5}{2}\right)$

Reason:

Find M, midpoint of AB:

$$M = \left(\frac{2+8}{2},\ \frac{6+4}{2}\right) = (5,\ 5)$$

Find N, midpoint of BC:

$$N = \left(\frac{8+4}{2},\ \frac{4+0}{2}\right) = (6,\ 2)$$

Find midpoint of MN:

$$\text{Midpoint of } MN = \left(\frac{5+6}{2},\ \frac{5+2}{2}\right) = \left(\frac{11}{2},\ \frac{7}{2}\right)$$

Corrected Answer: (C) $\left(\frac{11}{2},\ \frac{7}{2}\right)$

Q11. Correct Option: (B) B coincides with M, confirming that A, B, C are collinear and B is the midpoint of AC.

Reason: M divides AC in ratio $1:1$, so M is the midpoint of AC:

$$M = \left(\frac{1+5}{2},\ \frac{1+9}{2}\right) = (3,\ 5)$$

Since $B = (3,\ 5) = M$, point B coincides with the midpoint of AC. This confirms A, B, C are collinear and B is the midpoint of AC.

Section B — Short Answer Questions

Q12. *Formula used (Internal Section Formula):* The point dividing $A(x_1, y_1)$ and $B(x_2, y_2)$ in ratio $m:n$ internally is:

$$\left(\frac{mx_2 + nx_1}{m+n},\ \frac{my_2 + ny_1}{m+n}\right)$$

Given: $A(-4,\ 6)$, $B(8,\ -3)$.

Case 1 — Ratio $2:1$:

$$x = \frac{2(8) + 1(-4)}{2+1} = \frac{16-4}{3} = \frac{12}{3} = 4$$

$$y = \frac{2(-3) + 1(6)}{3} = \frac{-6+6}{3} = \frac{0}{3} = 0$$

$$\text{Point dividing in ratio } 2:1 = (\mathbf{4},\ \mathbf{0})$$

Case 2 — Ratio $1:2$:

$$x = \frac{1(8) + 2(-4)}{1+2} = \frac{8-8}{3} = \frac{0}{3} = 0$$

$$y = \frac{1(-3) + 2(6)}{3} = \frac{-3+12}{3} = \frac{9}{3} = 3$$

$$\text{Point dividing in ratio } 1:2 = (\mathbf{0},\ \mathbf{3})$$

Q13. Given: $M(3,\ -1)$ is the midpoint of $P(a+2,\ b-3)$ and $Q(2a-4,\ b+5)$.

Use the midpoint formula for the x-coordinate:

$$\frac{(a+2) + (2a-4)}{2} = 3$$

$$\frac{3a-2}{2} = 3$$

$$3a - 2 = 6$$

$$3a = 8$$

$$a = \frac{8}{3}$$

Use the midpoint formula for the y-coordinate:

$$\begin{aligned}\frac{(b-3)+(b+5)}{2} &= -1\\ \frac{2b+2}{2} &= -1\\ b+1 &= -1\\ b &= -2\end{aligned}$$

Co-ordinates of P and Q:

$$P = (a+2,\ b-3) = \left(\frac{8}{3}+2,\ -2-3\right) = \left(\frac{14}{3},\ -5\right)$$

$$Q = (2a-4,\ b+5) = \left(\frac{16}{3}-4,\ -2+5\right) = \left(\frac{4}{3},\ 3\right)$$

Verification:

$$\text{Midpoint} = \left(\frac{\frac{14}{3}+\frac{4}{3}}{2},\ \frac{-5+3}{2}\right) = \left(\frac{\frac{18}{3}}{2},\ \frac{-2}{2}\right) = (3,\ -1) \quad \checkmark$$

Q14. Given: $P(2k, 6)$, $Q(1, k-1)$, $R(k+2, -2k)$, centroid $G(3, 1)$.

Use x-coordinate of centroid:

$$\begin{aligned}\frac{2k+1+(k+2)}{3} &= 3\\ \frac{3k+3}{3} &= 3\\ k+1 &= 3\\ k &= 2\end{aligned}$$

Verify using y-coordinate of centroid:

$$\begin{aligned}\frac{6+(k-1)+(-2k)}{3} &= 1\\ \frac{6+k-1-2k}{3} &= 1\\ \frac{5-k}{3} &= 1\\ 5-k &= 3\\ k &= 2 \quad \checkmark\end{aligned}$$

Co-ordinates of the vertices with $k = 2$:

$$\begin{aligned}P &= (2(2),\ 6) = (\mathbf{4},\ \mathbf{6})\\ Q &= (1,\ 2-1) = (\mathbf{1},\ \mathbf{1})\\ R &= (2+2,\ -2(2)) = (\mathbf{4},\ -\mathbf{4})\end{aligned}$$

Verification: $G = \left(\dfrac{4+1+4}{3},\ \dfrac{6+1+(-4)}{3}\right) = \left(\dfrac{9}{3}, \dfrac{3}{3}\right) = (3,\ 1)$ ✓

Q15. Since Q lies on the y-axis, its x-coordinate is 0. Let Q divide $A(4,\ -5)$ and $B(-6,\ 10)$ internally in ratio $m : n$.

(i) **Finding the ratio:**
Using the x-coordinate condition:

$$\begin{aligned}\frac{m(-6)+n(4)}{m+n} &= 0\\ -6m+4n &= 0\\ 6m &= 4n\\ \frac{m}{n} &= \frac{4}{6} = \frac{2}{3}\\ \text{Ratio} &= \mathbf{2} : \mathbf{3}\end{aligned}$$

(ii) **Co-ordinates of Q:**
Using the y-coordinate with $m : n = 2 : 3$:

$$y_Q = \frac{2(10)+3(-5)}{2+3} = \frac{20-15}{5} = \frac{5}{5} = 1$$

$$Q = (\mathbf{0},\ \mathbf{1})$$

Q16. Given: Parallelogram $ABCD$ with $A(1,\ 2)$, $B(4,\ y)$, $C(x,\ 6)$, $D(3,\ 5)$.

Property used: The diagonals of a parallelogram bisect each other, so the midpoint of diagonal AC equals the midpoint of diagonal BD.

Midpoint of AC:

$$\text{Midpoint of } AC = \left(\frac{1+x}{2},\ \frac{2+6}{2}\right) = \left(\frac{1+x}{2},\ 4\right)$$

Midpoint of BD:

$$\text{Midpoint of } BD = \left(\frac{4+3}{2},\ \frac{y+5}{2}\right) = \left(\frac{7}{2},\ \frac{y+5}{2}\right)$$

Equate the midpoints:

$$\frac{1+x}{2} = \frac{7}{2} \implies 1+x = 7 \implies \boxed{x = 6}$$

$$4 = \frac{y+5}{2} \implies y+5 = 8 \implies \boxed{y = 3}$$

Verification: Midpoint of AC $= \left(\dfrac{1+6}{2},\ 4\right) = \left(\dfrac{7}{2},\ 4\right)$; Midpoint of BD $= \left(\dfrac{7}{2},\ \dfrac{3+5}{2}\right) = \left(\dfrac{7}{2},\ 4\right)$ ✓

Section C — Long Answer Questions

Q17. Given: $A(2,\ 5)$, $B(14,\ 11)$; P is twice as far from A as from B.

(i) **Ratio of internal division:**
Since $AP = 2 \cdot PB$, we have $AP : PB = 2 : 1$.
P divides AB internally in the ratio **2 : 1**.

(ii) **Co-ordinates of petrol station P:**

$$x_P = \frac{2(14)+1(2)}{2+1} = \frac{28+2}{3} = \frac{30}{3} = 10$$

$$y_P = \frac{2(11)+1(5)}{3} = \frac{22+5}{3} = \frac{27}{3} = 9$$

$$P = (\mathbf{10},\ \mathbf{9})$$

(iii) **Co-ordinates of health centre H (midpoint of PA):**

$$H = \left(\frac{10+2}{2},\ \frac{9+5}{2}\right) = \left(\frac{12}{2},\ \frac{14}{2}\right) = (\mathbf{6},\ \mathbf{7})$$

(iv) **Distance PB:**

$$\begin{aligned} PB &= \sqrt{(14-10)^2 + (11-9)^2} \\ &= \sqrt{4^2 + 2^2} \\ &= \sqrt{16+4} = \sqrt{20} = \mathbf{2\sqrt{5}} \text{ km} \end{aligned}$$

Q18. Given: $A(-2,\ 4)$, $B(6,\ -2)$, $C(2,\ 8)$.

(i) **Centroid G:**

$$G = \left(\frac{-2+6+2}{3},\ \frac{4+(-2)+8}{3}\right) = \left(\frac{6}{3},\ \frac{10}{3}\right) = \left(2,\ \frac{10}{3}\right)$$

(ii) **Midpoint M of BC (water tap):**

$$M = \left(\frac{6+2}{2},\ \frac{-2+8}{2}\right) = (4,\ 3)$$

(iii) **Point dividing median AM in ratio $2:1$ from A:**
$A(-2,\ 4)$, $M(4,\ 3)$, ratio $2 : 1$:

$$x = \frac{2(4)+1(-2)}{3} = \frac{8-2}{3} = \frac{6}{3} = 2$$

$$y = \frac{2(3)+1(4)}{3} = \frac{6+4}{3} = \frac{10}{3}$$

$$\text{Fencing post} = \left(2,\ \frac{10}{3}\right) = G \quad \checkmark$$

The fencing post **coincides** with the centroid G. This confirms the known result that the centroid divides each median in ratio $2 : 1$ from the vertex.

(iv) **Midpoints of AB and CA, and centroid of midpoint triangle:**

$$\text{Midpoint of } AB = \left(\frac{-2+6}{2},\ \frac{4+(-2)}{2}\right) = (2,\ 1)$$

$$\text{Midpoint of } CA = \left(\frac{2+(-2)}{2},\ \frac{8+4}{2}\right) = (0,\ 6)$$

$$M \text{ (midpoint of } BC) = (4,\ 3)$$

Centroid of the triangle formed by (2, 1), (0, 6), (4, 3):

$$= \left(\frac{2+0+4}{3},\ \frac{1+6+3}{3}\right) = \left(\frac{6}{3},\ \frac{10}{3}\right) = \left(2,\ \frac{10}{3}\right) = G \quad \checkmark$$

The centroid of the triangle formed by the three midpoints coincides exactly with G.

Q19. Given: $P(1,\ 7)$, $Q(5,\ 3)$, $R(9,\ 11)$.

(i) **Light 1 and Light 2:**

Light 1 — Midpoint of PQ:

$$L_1 = \left(\frac{1+5}{2},\ \frac{7+3}{2}\right) = (\mathbf{3},\ \mathbf{5})$$

Light 2 — Divides QR in ratio $3:1$:

$$x = \frac{3(9)+1(5)}{4} = \frac{27+5}{4} = \frac{32}{4} = 8$$

$$y = \frac{3(11)+1(3)}{4} = \frac{33+3}{4} = \frac{36}{4} = 9$$

$$L_2 = (\mathbf{8},\ \mathbf{9})$$

(ii) **Light 3 — Centroid of $\triangle PQR$:**

$$L_3 = \left(\frac{1+5+9}{3},\ \frac{7+3+11}{3}\right) = \left(\frac{15}{3},\ \frac{21}{3}\right) = (\mathbf{5},\ \mathbf{7})$$

(iii) **Light 4 — Divides PR in ratio $1:2$:**

$$x = \frac{1(9)+2(1)}{3} = \frac{9+2}{3} = \frac{11}{3}$$

$$y = \frac{1(11)+2(7)}{3} = \frac{11+14}{3} = \frac{25}{3}$$

$$L_4 = \left(\frac{11}{3},\ \frac{25}{3}\right)$$

(iv) **Verification — midpoint of L_1L_2 vs L_3:**

$$\text{Midpoint of } L_1(3,\ 5) \text{ and } L_2(8,\ 9) = \left(\frac{3+8}{2},\ \frac{5+9}{2}\right) = \left(\frac{11}{2},\ 7\right)$$

Since $L_3 = (5,\ 7)$ and $\dfrac{11}{2} = 5.5 \neq 5$, the midpoint of L_1L_2 is $\left(\dfrac{11}{2},\ 7\right) \neq (5,\ 7) = L_3$.

Conclusion: Light 3 does **not** coincide with the midpoint of the segment joining Light 1 and Light 2.

Q20. Given: $A(2,\ 1)$, $B(10,\ 5)$, $C(4,\ 9)$.

(i) **Midpoints D, E, F:**

$$D \text{ (midpoint of } BC) = \left(\frac{10+4}{2},\ \frac{5+9}{2}\right) = (7,\ 7)$$

$$E \text{ (midpoint of } CA) = \left(\frac{4+2}{2},\ \frac{9+1}{2}\right) = (3,\ 5)$$

$$F \text{ (midpoint of } AB) = \left(\frac{2+10}{2},\ \frac{1+5}{2}\right) = (6,\ 3)$$

(ii) **Centroid G of $\triangle ABC$:**

$$G = \left(\frac{2+10+4}{3},\ \frac{1+5+9}{3}\right) = \left(\frac{16}{3},\ \frac{15}{3}\right) = \left(\frac{16}{3},\ 5\right)$$

(iii) **Verification that G divides AD in ratio $2:1$:**
$A(2,\ 1)$, $D(7,\ 7)$, ratio $2:1$:

$$x = \frac{2(7)+1(2)}{3} = \frac{14+2}{3} = \frac{16}{3} \quad \checkmark$$

$$y = \frac{2(7)+1(1)}{3} = \frac{14+1}{3} = \frac{15}{3} = 5 \quad \checkmark$$

The section formula gives $\left(\dfrac{16}{3},\ 5\right) = G$. Hence G divides median AD in ratio $2:1$ from A. ✓

(iv) **Centroid of medial triangle DEF:**
$D(7,\ 7)$, $E(3,\ 5)$, $F(6,\ 3)$:

$$\text{Centroid of } \triangle DEF = \left(\frac{7+3+6}{3},\ \frac{7+5+3}{3}\right) = \left(\frac{16}{3},\ \frac{15}{3}\right) = \left(\frac{16}{3},\ 5\right) = G \quad \checkmark$$

The centroid of the medial triangle DEF coincides exactly with the centroid G of the original triangle ABC, illustrating that the centroid is invariant under the medial triangle construction.

Q21. Given: $A(a,\ b)$, $B(b,\ a)$, $C(a+b,\ a-b)$, with $a \neq b$, $a, b \neq 0$.

(i) **Centroid G in terms of a and b:**

$$G = \left(\frac{a+b+(a+b)}{3},\ \frac{b+a+(a-b)}{3}\right)$$
$$= \left(\frac{2a+2b}{3},\ \frac{2a}{3}\right) = \left(\frac{2(a+b)}{3},\ \frac{2a}{3}\right)$$

(ii) **Show P (dividing CM in ratio $2:1$) coincides with G:**
Midpoint M of AB:

$$M = \left(\frac{a+b}{2},\ \frac{b+a}{2}\right) = \left(\frac{a+b}{2},\ \frac{a+b}{2}\right)$$

P divides CM in ratio $2:1$ from $C(a+b,\ a-b)$:

$$x_P = \frac{2\cdot\dfrac{a+b}{2} + 1\cdot(a+b)}{3} = \frac{(a+b)+(a+b)}{3} = \frac{2(a+b)}{3}$$

$$y_P = \frac{2\cdot\dfrac{a+b}{2} + 1\cdot(a-b)}{3} = \frac{(a+b)+(a-b)}{3} = \frac{2a}{3}$$

$$P = \left(\frac{2(a+b)}{3},\ \frac{2a}{3}\right) = G \quad \checkmark$$

Hence P coincides with the centroid G.

(iii) **Finding b given G lies on $y = x + 2$:**
Substituting $G = \left(\dfrac{2(a+b)}{3},\ \dfrac{2a}{3}\right)$ into $y = x + 2$:

$$\frac{2a}{3} = \frac{2(a+b)}{3} + 2$$
$$\frac{2a}{3} - \frac{2(a+b)}{3} = 2$$
$$\frac{2a-2a-2b}{3} = 2$$
$$\frac{-2b}{3} = 2$$
$$-2b = 6$$
$$b = -3$$

$$\boxed{b = -3}$$

(iv) **Finding a, the vertices, and the centroid:**
Given $a + b = 6$ and $b = -3$:

$$a + (-3) = 6 \implies a = 9$$

Vertices:

$$
\begin{aligned}
A &= (a,\ b) = (9,\ -3)\\
B &= (b,\ a) = (-3,\ 9)\\
C &= (a+b,\ a-b) = (6,\ 12)
\end{aligned}
$$

Centroid:

$$G = \left(\frac{2(a+b)}{3},\ \frac{2a}{3}\right) = \left(\frac{2(6)}{3},\ \frac{2(9)}{3}\right) = (4,\ 6)$$

Verification that $G(4,6)$ lies on $y = x + 2$: $6 = 4 + 2$ ✓