Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (B) $\triangle ABC \sim \triangle PQR$ (similar) by AA criterion.

Reason: If two angles of one triangle are equal to two corresponding angles of another triangle, the triangles are similar by the **Angle-Angle (AA)** criterion. Since $\angle A = \angle P$ and $\angle B = \angle Q$, we get $\angle C = \angle R$ (angle sum property), so $\triangle ABC \sim \triangle PQR$.

Q2. Correct Option: (B) 6

Reason: By the Basic Proportionality Theorem (BPT), since $DE \parallel BC$:

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{3}{5} = \frac{AE}{10}$$
$$AE = \frac{3 \times 10}{5} = \mathbf{6}$$

Q3. Correct Option: (C) $9 : 25$

Reason: The ratio of areas of two similar triangles equals the **square** of the ratio of their corresponding sides.

$$\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

Q4. Correct Option: (B) 3 **cm**

Reason: Since $\triangle ABC \sim \triangle PQR$, corresponding sides are proportional.

$$\frac{AC}{PR} = \frac{AB}{PQ}$$
$$\frac{8}{4} = \frac{6}{x}$$
$$x = \frac{6 \times 4}{8} = \mathbf{3} \text{ cm}$$

Q5. Correct Option: (B) 150 **km^2**

Reason: Scale: 1 cm = 5 km. The area scale factor is the square of the length scale factor.

$$\text{Area scale factor} = 5^2 = 25 \text{ km}^2 \text{ per cm}^2$$

$$\text{Actual area} = 6 \times 25 = \mathbf{150} \text{ km}^{\mathbf{2}}$$

Q6. Correct Option: (C) SAS similarity

Reason: Two sides of one triangle are proportional to two corresponding sides of the other triangle $\left(\frac{AB}{DE} = \frac{AC}{DF} = \frac{1}{2}\right)$ and the **included angle** between them is equal ($\angle A = \angle D$). This satisfies the **SAS similarity** criterion.

Q7. Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Reason:

Assertion A is TRUE. All squares have all angles equal to 90° and all sides equal within each square, so the ratio of any two corresponding sides between two squares is constant. Both conditions for similarity (equal angles and proportional sides) are satisfied.

Reason R is TRUE and correctly explains why: equal corresponding angles (90° each) and proportional corresponding sides (ratio of their side lengths) together satisfy the definition of similar figures. R is the correct explanation of A.

Q8. Correct Option: (B) Congruent triangles are always similar, with a scale factor of 1.

Reason: Congruent triangles have all corresponding sides equal and all corresponding angles equal. This is a special case of similarity where the scale factor (ratio of corresponding sides) equals 1. Hence congruent triangles are **always** similar.

Q9. Correct Option: (B) No, because $\frac{PS}{SQ} \neq \frac{PT}{TR}$.

Reason: By the converse of BPT, $ST \parallel QR$ if and only if $\frac{PS}{SQ} = \frac{PT}{TR}$.

$$\frac{PS}{SQ} = \frac{4}{6} = \frac{2}{3}$$
$$\frac{PT}{TR} = \frac{5}{7}$$

Since $\frac{2}{3} \neq \frac{5}{7}$, the condition is **not** satisfied. Therefore ST is **not** parallel to QR.

Q10. Correct Option: (A) $\frac{84}{8}$ cm

Reason: The ratio of areas equals the square of the ratio of corresponding sides.

$$\frac{\text{Area}_{\text{small}}}{\text{Area}_{\text{large}}} = \left(\frac{s_{\text{small}}}{s_{\text{large}}}\right)^2$$
$$\frac{49}{64} = \left(\frac{s}{12}\right)^2$$
$$\frac{s}{12} = \frac{7}{8}$$
$$s = \frac{7 \times 12}{8} = \frac{84}{8} = \frac{21}{2} = \frac{63}{8} \times \frac{8}{8}$$
$$s = \frac{7 \times 12}{8} = \frac{84}{8} \approx 10.5 \text{ cm}$$

Q11. Correct Option: (C) 1,000,000,000 cm^3

Reason: Scale = 1 : 200. The volume scale factor is the **cube** of the length scale factor.

$$\text{Volume scale factor} = 200^3 = 8{,}000{,}000$$

$$\text{Actual volume} = 125 \times 8{,}000{,}000 = \mathbf{1{,}000{,}000{,}000}\ \text{cm}^{\mathbf{3}}$$

Section B — Short Answer Questions

Q12. Applying the Basic Proportionality Theorem (BPT):

Since $DE \parallel BC$:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{3x-1}{2x+1} = \frac{x+3}{x+5}$$

$$(3x-1)(x+5) = (x+3)(2x+1)$$

$$3x^2 + 15x - x - 5 = 2x^2 + x + 6x + 3$$

$$3x^2 + 14x - 5 = 2x^2 + 7x + 3$$

$$x^2 + 7x - 8 = 0$$

$$(x+8)(x-1) = 0$$

$x = 1$ (rejecting $x = -8$ as it gives negative lengths).

$$AD = 3(1) - 1 = \mathbf{2}\ \text{cm}$$

$$DB = 2(1) + 1 = \mathbf{3}\ \text{cm}$$

Verification: $\dfrac{AD}{DB} = \dfrac{2}{3}$ and $\dfrac{AE}{EC} = \dfrac{1+3}{1+5} = \dfrac{4}{6} = \dfrac{2}{3}$ ✓

Q13. Step 1 — Establish the similarity:

In $\triangle ACD$ and $\triangle ABC$: $\angle A$ is common, and $\angle ADC = \angle ACB = 90°$.

By AA criterion: $\triangle ACD \sim \triangle ABC$.

Step 2 — Use the altitude-on-hypotenuse result:

From $\triangle CBD \sim \triangle CAB$ (AA: $\angle B$ common, $\angle BDC = \angle BCA = 90°$):

$$\frac{BC}{AB} = \frac{DB}{BC} \implies BC^2 = DB \cdot AB$$

$$4^2 = DB \cdot (9 + DB)$$

$$16 = DB^2 + 9 \cdot DB$$

$$DB^2 + 9 \cdot DB - 16 = 0$$

Step 3 — Solve for DB:

$$DB = \frac{-9 + \sqrt{81 + 64}}{2} = \frac{-9 + \sqrt{145}}{2} \text{ cm} \approx 1.52 \text{ cm}$$

(Negative root rejected as length must be positive.)

Step 4 — Find AB:

$$AB = AD + DB = 9 + \frac{-9 + \sqrt{145}}{2} = \frac{9 + \sqrt{145}}{2} \text{ cm} \approx 10.52 \text{ cm}$$

Q14. Given: $\triangle ABC \sim \triangle DEF$, $AB = 6$ cm, $DE = 9$ cm.

(i) **Ratio of areas:**

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{6}{9}\right)^2 = \left(\frac{2}{3}\right)^2 = \mathbf{4 : 9}$$

(ii) **Area of $\triangle ABC$:**

$$\frac{\text{Area of } \triangle ABC}{108} = \frac{4}{9}$$

$$\text{Area of } \triangle ABC = \frac{4 \times 108}{9} = \mathbf{48} \text{ cm}^{\mathbf{2}}$$

(iii) **Length EF:**
Corresponding sides are in the ratio 2 : 3 (same as $AB : DE$).

$$\frac{BC}{EF} = \frac{2}{3}$$

$$EF = \frac{8 \times 3}{2} = \mathbf{12} \text{ cm}$$

Q15. Scale = 1 : 25,000, so 1 cm on map = 25,000 cm = 0.25 km actual.

(i) **Actual dimensions:**

$$\text{Length} = 4 \times 25{,}000 \text{ cm} = 1{,}00{,}000 \text{ cm} = \mathbf{1} \text{ km}$$

$$\text{Breadth} = 6 \times 25{,}000 \text{ cm} = 1{,}50{,}000 \text{ cm} = \mathbf{1.5} \text{ km}$$

(ii) **Actual area:**

$$\text{Area} = 1 \times 1.5 = \mathbf{1.5} \text{ km}^{\mathbf{2}}$$

Alternatively: Area scale factor = $(25{,}000)^2 = 6.25 \times 10^8$; Map area = $4 \times 6 = 24$ cm^2; Actual area = $24 \times 6.25 \times 10^8$ cm^2 = 1.5 km^2 ✓

(iii) **Map area for actual area 6.25 km^2:**

$$\text{Area on map} = \frac{\text{Actual area}}{\text{Area scale factor}} = \frac{6.25 \text{ km}^2}{(0.25)^2 \text{ km}^2/\text{cm}^2} = \frac{6.25}{0.0625} = \mathbf{100} \text{ cm}^{\mathbf{2}}$$

Q16. $\triangle ABC$: $AB = 6$, $BC = 8$, $CA = 10$. $\triangle PQR$: $PQ = 9$, $QR = 12$, $RP = 15$.

(i) **SSS verification:**

$$\frac{AB}{PQ} = \frac{6}{9} = \frac{2}{3}, \quad \frac{BC}{QR} = \frac{8}{12} = \frac{2}{3}, \quad \frac{CA}{RP} = \frac{10}{15} = \frac{2}{3}$$

Since $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{2}{3}$, all three pairs of corresponding sides are proportional.

By the **SSS similarity criterion**: $\triangle ABC \sim \triangle PQR$ with ratio of similarity $= \mathbf{2 : 3}$.

(ii) **Ratio of perimeters:**

$$\frac{P(\triangle ABC)}{P(\triangle PQR)} = \frac{6+8+10}{9+12+15} = \frac{24}{36} = \mathbf{2 : 3}$$

(The ratio of perimeters equals the ratio of corresponding sides.)

(iii) **Ratio of areas:**

$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = \left(\frac{2}{3}\right)^2 = \mathbf{4 : 9}$$

Section C — Long Answer Questions

Q17. Scale 1 : 40,000, so 1 cm = 40,000 cm = 400 m = 0.4 km.

(i) **Actual distance between schools:**

$$\text{Actual distance} = 5 \times 40{,}000 \text{ cm} = 2{,}00{,}000 \text{ cm} = \mathbf{2\ km}$$

(ii) **Actual perimeter of triangular garden:**

$$\text{Map perimeter} = 3 + 4 + 5 = 12 \text{ cm}$$
$$\text{Actual perimeter} = 12 \times 40{,}000 \text{ cm} = 4{,}80{,}000 \text{ cm} = \mathbf{4{,}800\ m}$$

(iii) **Actual area of triangular garden:**
The triangle with sides 3, 4, 5 is right-angled (since $3^2 + 4^2 = 5^2$).

$$\text{Map area} = \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$$
$$\text{Area scale factor} = (40{,}000)^2 = (400)^2 \text{ m}^2/\text{cm}^2 = 1{,}60{,}000 \text{ m}^2/\text{cm}^2$$
$$\text{Actual area} = 6 \times 1{,}60{,}000 = \mathbf{9{,}60{,}000\ m^2}$$

(iv) **Cost of road construction:**

$$\text{Road on map} = 5 \text{ cm}$$
$$\text{Actual length} = 5 \times 400 = 2{,}000 \text{ m}$$
$$\text{Total cost} = 2{,}000 \times 80 = \mathbf{Rs}\ 1{,}60{,}000$$

Q18. Lamp post height = 6 m, shadow = 4 m; Tree shadow = 10 m.

(i) **Are the triangles similar?**

Both the lamp post and the tree are vertical. At the same instant, the sun's rays make the same angle with the ground, forming two right-angled triangles (right angle at the base of each object).

In $\triangle_{\text{lamp}}$ and $\triangle_{\text{tree}}$: $90°$ at the base of each, and equal angles of elevation of the sun. By **AA criterion**, the triangles are similar.

(ii) **Height of the tree:**

$$\frac{\text{Height of tree}}{\text{Shadow of tree}} = \frac{\text{Height of lamp}}{\text{Shadow of lamp}}$$

$$\frac{h}{10} = \frac{6}{4}$$

$$h = \frac{6 \times 10}{4} = \mathbf{15}\text{ m}$$

(iii) **Height of the building (shadow = 25 m):**

$$\frac{H}{25} = \frac{6}{4}$$

$$H = \frac{6 \times 25}{4} = \mathbf{37.5}\text{ m}$$

(iv) **New shadow of tree if lamp height = 4 m, lamp shadow = 4 m:**

New ratio $= \dfrac{\text{shadow}}{\text{height}} = \dfrac{4}{4} = 1$.

Tree height = 15 m (unchanged). New tree shadow:

$$\frac{\text{shadow}}{15} = 1 \implies \text{shadow} = \mathbf{15}\text{ m}$$

Q19. Trapezium $ABCD$, $AB \parallel DC$; $AB = 9$ cm, $DC = 6$ cm.

(i) **Proof that $\triangle AOB \sim \triangle COD$:**

In $\triangle AOB$ and $\triangle COD$:

$$\angle OAB = \angle OCD \quad \text{(alternate interior angles, } AB \parallel DC)$$
$$\angle OBA = \angle ODC \quad \text{(alternate interior angles, } AB \parallel DC)$$
$$\angle AOB = \angle COD \quad \text{(vertically opposite angles)}$$

By the **AA criterion**: $\triangle AOB \sim \triangle COD$.

(ii) **Ratios $OA : OC$ and $OB : OD$:**

Since $\triangle AOB \sim \triangle COD$, corresponding sides are proportional:

$$\frac{OA}{OC} = \frac{OB}{OD} = \frac{AB}{DC} = \frac{9}{6} = \mathbf{3 : 2}$$

(iii) **Finding OC and AC:**

$$\frac{OA}{OC} = \frac{3}{2}$$
$$\frac{4.5}{OC} = \frac{3}{2}$$
$$OC = \frac{4.5 \times 2}{3} = \mathbf{3\ cm}$$
$$AC = OA + OC = 4.5 + 3 = \mathbf{7.5\ cm}$$

(iv) **Area of $\triangle COD$:**

$$\frac{\text{Area}(\triangle AOB)}{\text{Area}(\triangle COD)} = \left(\frac{AB}{DC}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$
$$\text{Area}(\triangle COD) = \frac{27 \times 4}{9} = \mathbf{12\ cm}^2$$

Q20. $DE \parallel BC$; $AD = 3$, $DB = 4.5$, $EC = 6$, $BC = 10.5$.

(i) **Finding AE by BPT:**

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{3}{4.5} = \frac{AE}{6}$$
$$AE = \frac{3 \times 6}{4.5} = \mathbf{4}\ \text{cm}$$

(ii) **Showing $\triangle ADE \sim \triangle ABC$:**
In $\triangle ADE$ and $\triangle ABC$:

$$\angle A = \angle A \quad \text{(common)}$$
$$\angle ADE = \angle ABC \quad (\text{corresponding angles, } DE \parallel BC)$$

By **AA criterion**: $\triangle ADE \sim \triangle ABC$.

(iii) **Length of DE:**

$$AB = AD + DB = 3 + 4.5 = 7.5\ \text{cm}$$
$$\frac{DE}{BC} = \frac{AD}{AB} = \frac{3}{7.5} = \frac{2}{5}$$
$$DE = \frac{2}{5} \times 10.5 = \mathbf{4.2}\ \text{cm}$$

(iv) **Ratio of area $\triangle ADE$ to trapezium $DBCE$:**

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

Let Area $(\triangle ABC) = 25k$. Then Area $(\triangle ADE) = 4k$.

$$\text{Area of trapezium } DBCE = 25k - 4k = 21k$$
$$\therefore \quad \frac{\text{Area}(\triangle ADE)}{\text{Area}(DBCE)} = \frac{4k}{21k} = \mathbf{4 : 21}$$