Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (B) $\dfrac{12}{13}$

Reason: Using the identity $\sin^2 A + \cos^2 A = 1$:

$$\cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\cos A = \frac{12}{13} \quad \text{(taking positive value for acute angle } A\text{)}$$

Q2. Correct Option: (C) $\tan^2 A$

Reason: From the identity $1 + \tan^2 A = \sec^2 A$:

$$\sec^2 A - 1 = \tan^2 A$$

Q3. Correct Option: (D) 1

Reason: From the identity $1 + \cot^2 A = \csc^2 A$:

$$\csc^2 A - \cot^2 A = 1$$

Q4. Correct Option: (B) $\sec^2 A$

Reason: Using $\sin^2 A + \cos^2 A = 1$:

$$\frac{\sin^2 A + \cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A} = \sec^2 A$$

Q5. Correct Option: (A) $\dfrac{25}{16}$

Reason: Using the identity $\sec^2 A = 1 + \tan^2 A$:

$$\sec^2 A = 1 + \left(\frac{3}{4}\right)^2 = 1 + \frac{9}{16} = \frac{16}{16} + \frac{9}{16} = \frac{25}{16}$$

Q6. Correct Option: (A) $\dfrac{25}{24}$

Reason: Using the identity $\csc^2 A = 1 + \cot^2 A$:

$$\csc^2 A = 1 + \left(\frac{7}{24}\right)^2 = 1 + \frac{49}{576} = \frac{576}{576} + \frac{49}{576} = \frac{625}{576}$$

$$\csc A = \frac{25}{24} \quad \text{(taking positive value for acute angle } A\text{)}$$

Q7. Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Reason:

Checking Assertion A:

$$\begin{aligned}(\sin A + \cos A)^2 &= \sin^2 A + 2\sin A\cos A + \cos^2 A\\ &= (\sin^2 A + \cos^2 A) + 2\sin A\cos A\\ &= 1 + 2\sin A\cos A\end{aligned}$$

Assertion A is TRUE.

Reason R correctly expands the square using the algebraic identity $(x+y)^2 = x^2 + 2xy + y^2$ and then applies $\sin^2 A + \cos^2 A = 1$. **R is TRUE** and provides the exact steps that establish A. Hence R is the correct explanation of A.

Q8. Correct Option: (C) $\cot^2 A - \csc^2 A = 1$

Reason: The correct identity is $1 + \cot^2 A = \csc^2 A$, which can be rearranged as:

$$\csc^2 A - \cot^2 A = 1$$

Option (C) states $\cot^2 A - \csc^2 A = 1$, which gives $-1 = 1$ (a contradiction). This is **not** a valid identity.

Verification of the other options:

$$\begin{aligned}&\text{(A)}: && \sin^2 A + \cos^2 A = 1 \quad \checkmark\\ &\text{(B)}: && 1 + \tan^2 A = \sec^2 A \quad \checkmark\\ &\text{(D)}: && 1 + \cot^2 A = \csc^2 A \quad \checkmark\end{aligned}$$

Q9. Correct Option: (D) 1

Reason: Simplifying each term separately:

$$\frac{\tan^2 A}{\sec^2 A} = \frac{\tan^2 A}{\sec^2 A} = \tan^2 A \cdot \cos^2 A = \frac{\sin^2 A}{\cos^2 A} \cdot \cos^2 A = \sin^2 A$$

$$\frac{\cot^2 A}{\csc^2 A} = \cot^2 A \cdot \sin^2 A = \frac{\cos^2 A}{\sin^2 A} \cdot \sin^2 A = \cos^2 A$$

Adding:

$$\frac{\tan^2 A}{\sec^2 A} + \frac{\cot^2 A}{\csc^2 A} = \sin^2 A + \cos^2 A = 1$$

Q10. Correct Option: (D) 1

Reason: Given $\sin A + \sin^2 A = 1$:

$$\sin A = 1 - \sin^2 A = \cos^2 A$$

Now evaluate $\cos^2 A + \cos^4 A$:

$$\begin{aligned}\cos^2 A + \cos^4 A &= \cos^2 A + (\cos^2 A)^2\\ &= \sin A + \sin^2 A \qquad (\text{substituting } \cos^2 A = \sin A)\\ &= 1 \qquad (\text{given condition})\end{aligned}$$

Q11. Correct Option: (B) Correct, and follows from the identity $\sec^2 A - \tan^2 A = 1$.

Reason: Using the difference of squares:

$$(\sec A - \tan A)(\sec A + \tan A) = \sec^2 A - \tan^2 A$$

From the identity $1 + \tan^2 A = \sec^2 A$:

$$\sec^2 A - \tan^2 A = 1$$

Therefore $(\sec A - \tan A)(\sec A + \tan A) = 1$ is a valid identity, true for all values in the domain where $\sec A$ and $\tan A$ are defined (i.e., $A \neq 90°$). It is not restricted to $A = 45°$ only.

Section B — Short Answer Questions

Q12. Part 1: Prove that $\cos^4 A - \sin^4 A = 1 - 2\sin^2 A$.

LHS:

$$\begin{aligned}
\cos^4 A - \sin^4 A &= (\cos^2 A)^2 - (\sin^2 A)^2 \\
&= (\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A) \quad [\text{using } a^2 - b^2 = (a+b)(a-b)] \\
&= 1 \cdot (\cos^2 A - \sin^2 A) \quad [\text{using } \sin^2 A + \cos^2 A = 1] \\
&= (1 - \sin^2 A) - \sin^2 A \quad [\text{substituting } \cos^2 A = 1 - \sin^2 A] \\
&= 1 - 2\sin^2 A \quad = \textbf{RHS}\checkmark
\end{aligned}$$

Part 2: Prove that $\dfrac{\tan A + \sin A}{\tan A - \sin A} = \dfrac{\sec A + 1}{\sec A - 1}$.

LHS:

$$\begin{aligned}
\frac{\tan A + \sin A}{\tan A - \sin A} &= \frac{\dfrac{\sin A}{\cos A} + \sin A}{\dfrac{\sin A}{\cos A} - \sin A} \quad [\text{writing } \tan A = \tfrac{\sin A}{\cos A}] \\
&= \frac{\sin A\left(\dfrac{1}{\cos A} + 1\right)}{\sin A\left(\dfrac{1}{\cos A} - 1\right)} \quad [\text{factoring out } \sin A] \\
&= \frac{\dfrac{1}{\cos A} + 1}{\dfrac{1}{\cos A} - 1} \quad [\text{cancelling } \sin A \neq 0] \\
&= \frac{\sec A + 1}{\sec A - 1} \quad [\text{since } \tfrac{1}{\cos A} = \sec A] \\
&= \textbf{RHS}\checkmark
\end{aligned}$$

Q13. Prove that $(\sec A - \tan A)^2 = \dfrac{1-\sin A}{1+\sin A}$**.**

LHS:

$$\begin{aligned}
(\sec A - \tan A)^2 &= \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2 \quad \text{[expressing in terms of } \sin A, \cos A\text{]} \\
&= \left(\frac{1-\sin A}{\cos A}\right)^2 \\
&= \frac{(1-\sin A)^2}{\cos^2 A} \\
&= \frac{(1-\sin A)^2}{1-\sin^2 A} \quad \text{[using } \cos^2 A = 1-\sin^2 A\text{]} \\
&= \frac{(1-\sin A)^2}{(1-\sin A)(1+\sin A)} \quad \text{[factoring the denominator]} \\
&= \frac{1-\sin A}{1+\sin A} \quad \text{[cancelling } (1-\sin A)\text{]} \\
&= \textbf{RHS}\checkmark
\end{aligned}$$

Q14. Prove that $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \dfrac{\csc A - 1}{\csc A + 1}$**.**

LHS:

$$\begin{aligned}
\frac{\cot A - \cos A}{\cot A + \cos A} &= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} \quad \text{[writing } \cot A = \tfrac{\cos A}{\sin A}\text{]} \\
&= \frac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)} \quad \text{[factoring out } \cos A\text{]} \\
&= \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1} \quad \text{[cancelling } \cos A \neq 0\text{]} \\
&= \frac{\csc A - 1}{\csc A + 1} \quad \text{[since } \tfrac{1}{\sin A} = \csc A\text{]} \\
&= \textbf{RHS}\checkmark
\end{aligned}$$

Q15. Given: $\sin A - \cos A = \dfrac{1}{\sqrt{2}}$, A is acute.

Step 1 — Find $\sin A + \cos A$**:**

Squaring the given condition:

$$
\begin{aligned}
(\sin A - \cos A)^2 &= \left(\frac{1}{\sqrt{2}}\right)^2 \\
\sin^2 A - 2\sin A\cos A + \cos^2 A &= \frac{1}{2} \\
1 - 2\sin A\cos A &= \frac{1}{2} \quad [\text{using } \sin^2 A + \cos^2 A = 1] \\
2\sin A\cos A &= \frac{1}{2} \quad \cdots (1)
\end{aligned}
$$

Now using:

$$
\begin{aligned}
(\sin A + \cos A)^2 &= \sin^2 A + 2\sin A\cos A + \cos^2 A \\
&= 1 + 2\sin A\cos A \\
&= 1 + \frac{1}{2} = \frac{3}{2}
\end{aligned}
$$

$$\sin A + \cos A = \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

(positive since A is acute and $\sin A + \cos A > 0$)

$$\boxed{\sin A + \cos A = \frac{\sqrt{6}}{2}}$$

Step 2 — Evaluate $\sin^3 A + \cos^3 A$:
Using the identity $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$:

$$
\begin{aligned}
\sin^3 A + \cos^3 A &= (\sin A + \cos A)(\sin^2 A - \sin A\cos A + \cos^2 A) \\
&= (\sin A + \cos A)(1 - \sin A\cos A) \\
&= \frac{\sqrt{6}}{2} \times \left(1 - \frac{1}{4}\right) \quad \left[\text{from (1): } \sin A\cos A = \frac{1}{4}\right] \\
&= \frac{\sqrt{6}}{2} \times \frac{3}{4} \\
&= \frac{3\sqrt{6}}{8}
\end{aligned}
$$

$$\boxed{\sin^3 A + \cos^3 A = \frac{3\sqrt{6}}{8}}$$

Solutions

Section C — Long Answer Questions

Q17. Right triangle ABC, right-angled at B; $AC = 25$, $BC = 7$.

(i) **Finding AB and the trigonometric ratios:**
By the Pythagorean theorem:

$$AB = \sqrt{AC^2 - BC^2} = \sqrt{625 - 49} = \sqrt{576} = 24 \text{ units}$$

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$$
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$
$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AB} = \frac{7}{24}$$

(ii) **Verification using $\sin^2 A + \cos^2 A = 1$:**

$$\sin^2 A + \cos^2 A = \left(\frac{7}{25}\right)^2 + \left(\frac{24}{25}\right)^2 = \frac{49}{625} + \frac{576}{625} = \frac{625}{625} = 1 \quad \checkmark$$

(iii) **Finding $\sec A$ using $1 + \tan^2 A = \sec^2 A$:**

$$\sec^2 A = 1 + \tan^2 A = 1 + \left(\frac{7}{24}\right)^2 = 1 + \frac{49}{576} = \frac{625}{576}$$
$$\sec A = \frac{25}{24}$$

From the triangle: $\sec A = \frac{AC}{AB} = \frac{25}{24}$ $\checkmark$

(iv) **Evaluating $\frac{\csc^2 A - \sec^2 A}{\csc^2 A + \sec^2 A}$:**

$$\csc A = \frac{25}{7} \implies \csc^2 A = \frac{625}{49}$$
$$\sec A = \frac{25}{24} \implies \sec^2 A = \frac{625}{576}$$

$$\csc^2 A - \sec^2 A = \frac{625}{49} - \frac{625}{576} = 625\left(\frac{576 - 49}{49 \times 576}\right) = 625 \times \frac{527}{28224}$$
$$\csc^2 A + \sec^2 A = 625\left(\frac{576 + 49}{28224}\right) = 625 \times \frac{625}{28224}$$

$$\frac{\csc^2 A - \sec^2 A}{\csc^2 A + \sec^2 A} = \frac{625 \times 527}{625 \times 625} = \frac{527}{625}$$

Q18. Given: $p = \frac{\sin A}{1 + \cos A}, \quad q = \frac{\sin A}{1 - \cos A}$.

(i) **Show $p \times q = 1$:**

$$p \times q = \frac{\sin A}{1+\cos A} \times \frac{\sin A}{1-\cos A} = \frac{\sin^2 A}{(1+\cos A)(1-\cos A)}$$
$$= \frac{\sin^2 A}{1-\cos^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1 \quad \checkmark$$

(ii) **Show $p + q = 2\csc A$:**

$$p + q = \frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A}$$
$$= \sin A \cdot \frac{(1-\cos A)+(1+\cos A)}{(1+\cos A)(1-\cos A)}$$
$$= \sin A \cdot \frac{2}{1-\cos^2 A} = \sin A \cdot \frac{2}{\sin^2 A} = \frac{2}{\sin A} = 2\csc A \quad \checkmark$$

(iii) **Verify $p - q = -\dfrac{2\cos A}{\sin A}$:**

$$p - q = \frac{\sin A}{1+\cos A} - \frac{\sin A}{1-\cos A}$$
$$= \sin A \cdot \frac{(1-\cos A)-(1+\cos A)}{(1-\cos^2 A)}$$
$$= \sin A \cdot \frac{-2\cos A}{\sin^2 A} = \frac{-2\cos A}{\sin A} \quad \checkmark$$

(iv) **With $\cos A = \dfrac{4}{5}$:**

$$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$$

$$p = \frac{\frac{3}{5}}{1+\frac{4}{5}} = \frac{\frac{3}{5}}{\frac{9}{5}} = \frac{1}{3}$$
$$q = \frac{\frac{3}{5}}{1-\frac{4}{5}} = \frac{\frac{3}{5}}{\frac{1}{5}} = 3$$

Verification of (i): $p \times q = \dfrac{1}{3} \times 3 = 1$ ✓

Verification of (ii): $p + q = \dfrac{1}{3} + 3 = \dfrac{10}{3}$; $2\csc A = \dfrac{2}{\frac{3}{5}} = \dfrac{10}{3}$ ✓

Q19. Given: $E = \dfrac{\tan^2 A}{\tan^2 A + 1} + \dfrac{\cot^2 A}{\cot^2 A + 1}$

(i) **Rewriting using standard identities:**

Using $\tan^2 A + 1 = \sec^2 A$ and $\cot^2 A + 1 = \csc^2 A$:

$$\begin{aligned}
E &= \frac{\tan^2 A}{\sec^2 A} + \frac{\cot^2 A}{\csc^2 A} \\
&= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{1}{\cos^2 A}} + \frac{\frac{\cos^2 A}{\sin^2 A}}{\frac{1}{\sin^2 A}} \\
&= \sin^2 A + \cos^2 A
\end{aligned}$$

(ii) **Simplified value and type:**

$$E = \sin^2 A + \cos^2 A = \mathbf{1}$$

E is a **constant** expression; its value is 1 regardless of the angle A.

(iii) **Verification for $A = 30°$:**

$$\tan 30° = \frac{1}{\sqrt{3}}, \quad \cot 30° = \sqrt{3}$$

$$E = \frac{\left(\frac{1}{\sqrt{3}}\right)^2}{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} + \frac{(\sqrt{3})^2}{(\sqrt{3})^2 + 1} = \frac{\frac{1}{3}}{\frac{1}{3} + 1} + \frac{3}{3+1} = \frac{\frac{1}{3}}{\frac{4}{3}} + \frac{3}{4} = \frac{1}{4} + \frac{3}{4} = 1 \quad \checkmark$$

(iv) **Justification that $E = \sin^2 A + \cos^2 A$ is a correct intermediate step:**

$$\begin{aligned}
E &= \frac{\tan^2 A}{\tan^2 A + 1} + \frac{\cot^2 A}{\cot^2 A + 1} \\
&= \frac{\tan^2 A}{\sec^2 A} + \frac{\cot^2 A}{\csc^2 A} \quad [\text{using } 1 + \tan^2 A = \sec^2 A,\ 1 + \cot^2 A = \csc^2 A] \\
&= \tan^2 A \cdot \cos^2 A + \cot^2 A \cdot \sin^2 A \quad \left[\text{since } \frac{1}{\sec^2 A} = \cos^2 A,\ \frac{1}{\csc^2 A} = \sin^2 A\right] \\
&= \frac{\sin^2 A}{\cos^2 A} \cdot \cos^2 A + \frac{\cos^2 A}{\sin^2 A} \cdot \sin^2 A \quad \left[\text{writing } \tan A = \frac{\sin A}{\cos A},\ \cot A = \frac{\cos A}{\sin A}\right] \\
&= \sin^2 A + \cos^2 A \quad \checkmark
\end{aligned}$$

The intermediate step is correct. The final value is 1 by the Pythagorean identity.

Q20. (i) **Prove:** $\dfrac{\sin A}{1 - \cot A} + \dfrac{\cos A}{1 - \tan A} = \sin A + \cos A$

LHS:

$$= \frac{\sin A}{1 - \frac{\cos A}{\sin A}} + \frac{\cos A}{1 - \frac{\sin A}{\cos A}}$$

$$= \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}} + \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\sin A - \cos A} + \frac{\cos^2 A}{\cos A - \sin A}$$

$$= \frac{\sin^2 A}{\sin A - \cos A} - \frac{\cos^2 A}{\sin A - \cos A}$$

$$= \frac{\sin^2 A - \cos^2 A}{\sin A - \cos A}$$

$$= \frac{(\sin A - \cos A)(\sin A + \cos A)}{\sin A - \cos A} \quad [\text{using } a^2 - b^2 = (a-b)(a+b)]$$

$$= \sin A + \cos A \quad = \textbf{RHS}\checkmark$$

(ii) **Prove:** $\dfrac{1}{1+\cot^2 A} + \dfrac{1}{1+\tan^2 A} = 1$

LHS:

$$= \frac{1}{\csc^2 A} + \frac{1}{\sec^2 A} \quad [\text{using } 1+\cot^2 A = \csc^2 A,\ 1+\tan^2 A = \sec^2 A]$$

$$= \sin^2 A + \cos^2 A$$

$$= 1 \quad = \textbf{RHS}\checkmark$$

(iii) **Express $\sin A$ in terms of k, given $\sec A + \tan A = k$:**

From the identity $\sec^2 A - \tan^2 A = 1$:

$$(\sec A + \tan A)(\sec A - \tan A) = 1$$

$$k(\sec A - \tan A) = 1$$

$$\sec A - \tan A = \frac{1}{k} \quad \cdots (2)$$

Adding $\sec A + \tan A = k$ and equation (2):

$$2\sec A = k + \frac{1}{k} = \frac{k^2+1}{k}$$

$$\sec A = \frac{k^2+1}{2k} \implies \cos A = \frac{2k}{k^2+1}$$

Subtracting equation (2) from $\sec A + \tan A = k$:

$$2\tan A = k - \frac{1}{k} = \frac{k^2-1}{k}$$

$$\tan A = \frac{k^2-1}{2k}$$

Using $\sin A = \tan A \cdot \cos A$:

$$\sin A = \frac{k^2-1}{2k} \times \frac{2k}{k^2+1}$$

$$\boxed{\sin A = \frac{k^2-1}{k^2+1}}$$

(iv) **Show** $\cos A = \dfrac{2k}{k^2+1}$**:**

From part (iii), $\cos A = \dfrac{1}{\sec A}$ and $\sec A = \dfrac{k^2+1}{2k}$:

$$\cos A = \frac{2k}{k^2+1} \quad \checkmark$$

Verification: $\sin^2 A + \cos^2 A = \left(\dfrac{k^2-1}{k^2+1}\right)^2 + \left(\dfrac{2k}{k^2+1}\right)^2 = \dfrac{(k^2-1)^2+4k^2}{(k^2+1)^2} = \dfrac{k^4+2k^2+1}{(k^2+1)^2} = 1$ ✓

Q21. (i) **Prove:** $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1+\sin A}{\cos A}$

LHS: Replace $1 = \sec^2 A - \tan^2 A = (\sec A - \tan A)(\sec A + \tan A)$ in the numerator:

$$= \frac{\tan A + \sec A - (\sec A - \tan A)(\sec A + \tan A)}{\tan A - \sec A + 1}$$

$$= \frac{(\sec A + \tan A)\big[1 - (\sec A - \tan A)\big]}{\tan A - \sec A + 1}$$

$$= \frac{(\sec A + \tan A)(1 - \sec A + \tan A)}{-(\ \sec A - \tan A - 1)}$$

$$= \frac{(\sec A + \tan A)(\tan A - \sec A + 1)}{(\tan A - \sec A + 1)}$$

$$= \sec A + \tan A$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \frac{1+\sin A}{\cos A} \quad = \textbf{RHS}\checkmark$$

(ii) **Prove:** $(\csc A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

LHS:

$$\csc A - \sin A = \frac{1}{\sin A} - \sin A = \frac{1-\sin^2 A}{\sin A} = \frac{\cos^2 A}{\sin A}$$

$$\sec A - \cos A = \frac{1}{\cos A} - \cos A = \frac{1-\cos^2 A}{\cos A} = \frac{\sin^2 A}{\cos A}$$

$$\tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} = \frac{1}{\sin A \cos A}$$

Multiplying all three:

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} \times \frac{1}{\sin A \cos A}$$

$$= \frac{\cos^2 A \cdot \sin^2 A}{\sin A \cdot \cos A \cdot \sin A \cdot \cos A} = \frac{\sin^2 A \cos^2 A}{\sin^2 A \cos^2 A} = 1 \quad = \textbf{RHS}\checkmark$$

(iii) **Prove:** $\dfrac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \sin A \cos A = 1$

LHS:

$$= \frac{(\sin A + \cos A)(\sin^2 A - \sin A \cos A + \cos^2 A)}{\sin A + \cos A} + \sin A \cos A$$

[using $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$]

$$= \sin^2 A - \sin A \cos A + \cos^2 A + \sin A \cos A$$

$$= \sin^2 A + \cos^2 A$$

$$= 1 \quad = \textbf{RHS}\checkmark$$

(iv) **Prove:** $\dfrac{\tan^3 A - 1}{\tan A - 1} = \sec^2 A + \tan A$

LHS:

$$\frac{\tan^3 A - 1}{\tan A - 1} = \frac{(\tan A - 1)(\tan^2 A + \tan A + 1)}{\tan A - 1} \quad \text{[using } a^3 - b^3 = (a-b)(a^2 + ab + b^2)\text{]}$$

$$= \tan^2 A + \tan A + 1$$

$$= (1 + \tan^2 A) + \tan A \quad \text{[rearranging]}$$

$$= \sec^2 A + \tan A \quad \text{[using } 1 + \tan^2 A = \sec^2 A\text{]}$$

$$= \textbf{RHS}\checkmark$$