Solutions

Section A — Multiple Choice Questions

Q1. Correct Option: (A) $\cos 17^\circ$

Reason: Using the complementary angle identity $\sin\theta = \cos(90^\circ - \theta)$:

$$\sin 73^\circ = \cos(90^\circ - 73^\circ) = \cos 17^\circ$$

17° lies between 0° and 45°. ✓

Q2. Correct Option: (B) $\tan 22^\circ$

Reason: Using the complementary angle identity $\cot\theta = \tan(90^\circ - \theta)$:

$$\cot 68^\circ = \tan(90^\circ - 68^\circ) = \tan 22^\circ$$

22° lies between 0° and 45°. ✓

Q3. Correct Option: (C) $\csc 32^\circ$

Reason: Using the complementary angle identity $\sec\theta = \csc(90^\circ - \theta)$:

$$\sec 58^\circ = \csc(90^\circ - 58^\circ) = \csc 32^\circ$$

Q4. Correct Option: (C) 2

$\sin 42^\circ\ \sec 48^\circ + \cos 42^\circ\ \csc 48^\circ = 2$

Note $48^\circ = 90^\circ - 42^\circ$. Applying complementary identities:

$$\sec 48^\circ = \sec(90^\circ - 42^\circ) = \csc 42^\circ$$
$$\csc 48^\circ = \csc(90^\circ - 42^\circ) = \sec 42^\circ$$

$$\begin{aligned} \text{LHS} &= \sin 42^\circ \cdot \csc 42^\circ + \cos 42^\circ \cdot \sec 42^\circ \\ &= \sin 42^\circ \cdot \frac{1}{\sin 42^\circ} + \cos 42^\circ \cdot \frac{1}{\cos 42^\circ} \\ &= 1 + 1 = 2 \end{aligned}$$

Q5. Correct Option: (C)

Reason: Given $\cos A = \dfrac{3}{5}$.

$$\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$
$$\tan A = \frac{\sin A}{\cos A} = \frac{4/5}{3/5} = \frac{4}{3}$$

$$5\sin A - 4\tan A = 5 \cdot \frac{4}{5} - 4 \cdot \frac{4}{3} = 4 - \frac{16}{3} = \frac{12 - 16}{3} = -\frac{4}{3}$$

Q6. Correct Option: (C) $\cos 31° + \cot 27°$

Reason: Using complementary angle identities:

$$\sin 59° = \cos(90° - 59°) = \cos 31°$$
$$\tan 63° = \cot(90° - 63°) = \cot 27°$$

$$\therefore \quad \sin 59° + \tan 63° = \cos 31° + \cot 27°$$

Q7. Correct Option: (A) Both A and R are true, and R is the correct explanation of A.

Reason:

Checking Assertion A:

Using complementary identities:

$$\sec 64° = \csc(90° - 64°) = \csc 26°$$
$$\csc 64° = \sec(90° - 64°) = \sec 26°$$

$$\begin{aligned}\frac{\sin 26°}{\sec 64°} + \frac{\cos 26°}{\csc 64°} &= \frac{\sin 26°}{\csc 26°} + \frac{\cos 26°}{\sec 26°} \\ &= \sin 26° \cdot \sin 26° + \cos 26° \cdot \cos 26° \\ &= \sin^2 26° + \cos^2 26° = 1\end{aligned}$$

Assertion A is TRUE. Reason R correctly describes every step used to establish A, so **R is TRUE and is the correct explanation of A.**

Q8. Correct Option: (B) Correct, since $90° - 48° = 42°$ **and** $\sec(90° - \theta) = \csc\theta$**.**

Reason: The student's working is valid:

$$\sec 48° = \sec(90° - 42°) = \csc 42° \quad [\text{complementary identity: } \sec(90° - \theta) = \csc\theta]$$

$$\sin 42° \cdot \csc 42° = 1 \quad [\text{since } \sin\theta \cdot \csc\theta = 1]$$

Every step is correct. The working is **valid**.

Q9. Correct Option: (A) 0

Reason:

Numerator:

Since $35° + 55° = 90°$: $\tan 55° = \cot 35°$, so $\tan 35° \cdot \tan 55° = \tan 35° \cdot \cot 35° = 1$.

Since $20° + 70° = 90°$: $\csc 70° = \sec 20° = \dfrac{1}{\cos 20°}$, so $\cos 20° \cdot \csc 70° = \cos 20° \cdot \dfrac{1}{\cos 20°} = 1$.

$$\text{Numerator} = 1 - 1 = 0$$

Denominator:

$$\sin^2 30^\circ + \sin^2 60^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

$$\therefore \quad \frac{0}{1} = \mathbf{0}$$

Q10. Correct Option: (A) $\sec 22^\circ + \tan 18^\circ$

Reason: Using complementary angle identities:

$$\csc 68^\circ = \csc(90^\circ - 22^\circ) = \sec 22^\circ$$
$$\cot 72^\circ = \cot(90^\circ - 18^\circ) = \tan 18^\circ$$

$$\csc 68^\circ + \cot 72^\circ = \sec 22^\circ + \tan 18^\circ$$

Both 22° and 18° lie between 0° and 45°. ✓

Q11. Correct Option: (c) 0

Reason: The product includes $\cos 90^\circ$ as one of its factors.

$$\cos 90^\circ = 0$$

Since any product containing a factor of 0 equals 0:

$$\cos 1^\circ \cdot \cos 2^\circ \cdots \cos 90^\circ \cdots \cos 100^\circ = 0$$

Section B — Short Answer Questions

Q12. *Key identities used:* $\sin\theta = \cos(90^\circ - \theta)$, $\tan\theta = \cot(90^\circ - \theta)$, $\sec\theta = \csc(90^\circ - \theta)$ and vice versa.

(i) $\csc 68^\circ + \cot 72^\circ$

$$\csc 68^\circ = \csc(90^\circ - 22^\circ) = \sec 22^\circ$$
$$\cot 72^\circ = \cot(90^\circ - 18^\circ) = \tan 18^\circ$$

$$\csc 68^\circ + \cot 72^\circ = \sec 22^\circ + \tan 18^\circ$$

(ii) $\cos 74^\circ + \sec 67^\circ$

$$\cos 74^\circ = \cos(90^\circ - 16^\circ) = \sin 16^\circ$$
$$\sec 67^\circ = \sec(90^\circ - 23^\circ) = \csc 23^\circ$$

$$\cos 74^\circ + \sec 67^\circ = \sin 16^\circ + \csc 23^\circ$$

(iii) $\sin 81^\circ - \tan 78^\circ + \cos 66^\circ$

$$\sin 81^\circ = \sin(90^\circ - 9^\circ) = \cos 9^\circ$$
$$\tan 78^\circ = \tan(90^\circ - 12^\circ) = \cot 12^\circ$$
$$\cos 66^\circ = \cos(90^\circ - 24^\circ) = \sin 24^\circ$$

$$\sin 81^\circ - \tan 78^\circ + \cos 66^\circ = \cos 9^\circ - \cot 12^\circ + \sin 24^\circ$$

Q13. Evaluate: $\left(\frac{\tan 60^\circ}{\sin 60^\circ}\right)^2 + \left(\frac{\cot 30^\circ}{\cos 30^\circ}\right)^2 - 3\left(\csc^2 45^\circ \cdot \sec^2 45^\circ\right)$

Standard values used:

$$\tan 60^\circ = \sqrt{3}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cot 30^\circ = \sqrt{3}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \csc 45^\circ = \sqrt{2}, \quad \sec 45^\circ = \sqrt{2}$$

Step 1:

$$\frac{\tan 60^\circ}{\sin 60^\circ} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}} = \sqrt{3} \cdot \frac{2}{\sqrt{3}} = 2 \implies \left(\frac{\tan 60^\circ}{\sin 60^\circ}\right)^2 = 4$$

Step 2:

$$\frac{\cot 30^\circ}{\cos 30^\circ} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}} = 2 \implies \left(\frac{\cot 30^\circ}{\cos 30^\circ}\right)^2 = 4$$

Step 3:

$$\csc^2 45^\circ \cdot \sec^2 45^\circ = (\sqrt{2})^2 \cdot (\sqrt{2})^2 = 2 \times 2 = 4$$
$$3\left(\csc^2 45^\circ \cdot \sec^2 45^\circ\right) = 3 \times 4 = 12$$

Step 4 — Final value:

$$4 + 4 - 12 = \mathbf{-4}$$

Q14. Given: $\frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

Step 1 — Divide numerator and denominator of LHS by $\cos x$:

$$\frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1 - \tan x}{1 + \tan x}$$

Step 2 — Equate with RHS:

$$\frac{1 - \tan x}{1 + \tan x} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$$

Comparing numerators and denominators directly:

$$\tan x = \sqrt{3}$$

Step 3 — Find x:

$$\tan x = \sqrt{3} \implies x = 60^\circ$$

Verification:

$$\frac{\cos 60^\circ - \sin 60^\circ}{\cos 60^\circ + \sin 60^\circ} = \frac{\frac{1}{2} - \frac{\sqrt{3}}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \quad \checkmark$$

$$\boxed{x = 60^\circ}$$

Section C — Long Answer Questions

Q15. Right triangle PQR, right-angled at Q; $PQ = 5$ cm, $PR = 13$ cm.

(i) **Finding QR and all trigonometric ratios:**

$$QR = \sqrt{PR^2 - PQ^2} = \sqrt{169-25} = \sqrt{144} = 12 \text{ cm}$$

$$\sin P = \frac{QR}{PR} = \frac{12}{13}, \quad \cos P = \frac{PQ}{PR} = \frac{5}{13}, \quad \tan P = \frac{QR}{PQ} = \frac{12}{5}$$

$$\csc P = \frac{13}{12}, \quad \sec P = \frac{13}{5}, \quad \cot P = \frac{5}{12}$$

(ii) **Verification of $\sin^2 P + \cos^2 P = 1$:**

$$\sin^2 P + \cos^2 P = \left(\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2 = \frac{144}{169} + \frac{25}{169} = \frac{169}{169} = 1 \quad \checkmark$$

(iii) **Showing $\sin P = \cos(90^\circ - P)$ numerically:**

$$\sin P = \frac{12}{13}$$

$$\cos(90^\circ - P) = \cos(90^\circ - P) = \frac{\text{adjacent to } (90^\circ - P)}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

Both sides equal $\frac{12}{13}$. ✓

(iv) **Evaluating $\dfrac{\sin P \ \csc P + \cos P \ \sec P}{\tan P + \cot P}$:**

Numerator: $\quad \sin P \cdot \csc P + \cos P \cdot \sec P = 1 + 1 = 2$

$$\text{Denominator:} \quad \tan P + \cot P = \frac{12}{5} + \frac{5}{12} = \frac{144+25}{60} = \frac{169}{60}$$

$$\frac{\sin P \ \csc P + \cos P \ \sec P}{\tan P + \cot P} = \frac{2}{\frac{169}{60}} = \frac{120}{169}$$

Q16. Ladder length $= 10$ m, $\sin A = \dfrac{4}{5}$.

(i) **Height and distance:**

$$\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

$$\text{Height on wall} = 10 \sin A = 10 \times \frac{4}{5} = 8 \text{ m}$$
$$\text{Distance from wall} = 10 \cos A = 10 \times \frac{3}{5} = 6 \text{ m}$$

(ii) **Angle with the wall and verification:**
In a right triangle, the two acute angles are complementary. The ladder makes angle A with the ground and angle $(90^\circ - A)$ with the wall.

$$\sin(90^\circ - A) = \cos A = \frac{3}{5}$$
$$\cos A = \frac{3}{5}$$

Both values equal $\dfrac{3}{5}$. ✓

(iii) **Finding $\cot A$ and $\csc A$:**
Using $1 + \cot^2 A = \csc^2 A$ and $\csc A = \dfrac{1}{\sin A} = \dfrac{5}{4}$:

$$\cot^2 A = \csc^2 A - 1 = \frac{25}{16} - 1 = \frac{9}{16}$$
$$\cot A = \frac{3}{4}$$

Verification from triangle: $\cot A = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{6}{8} = \dfrac{3}{4}$ ✓

$\csc A = \sqrt{1 + \cot^2 A} = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{25}{16}} = \dfrac{5}{4}$, which matches $\dfrac{1}{\sin A} = \dfrac{5}{4}$. ✓

(iv) **Evaluating $\dfrac{\sec A - \tan A}{\sec A + \tan A}$:**

$$\sec A = \frac{1}{\cos A} = \frac{5}{3}, \qquad \tan A = \frac{\sin A}{\cos A} = \frac{4}{3}$$

$$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\frac{5}{3} - \frac{4}{3}}{\frac{5}{3} + \frac{4}{3}} = \frac{\frac{1}{3}}{\frac{9}{3}} = \frac{1}{9}$$

Q17.

$$E = \frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 28^\circ + \cos^2 62^\circ} + \frac{\sin^2 63^\circ + \cos 63^\circ \sin 27^\circ}{\cos 27^\circ \sin 63^\circ + \cos^2 63^\circ}$$

(i) **Complementary angle relationships:**

$$\sin 68^\circ = \sin(90^\circ - 22^\circ) = \cos 22^\circ \quad [\text{using } \sin\theta = \cos(90^\circ - \theta)]$$
$$\cos 62^\circ = \cos(90^\circ - 28^\circ) = \sin 28^\circ \quad [\text{using } \cos\theta = \sin(90^\circ - \theta)]$$

(ii) **Simplifying the first fraction:**

$$\text{Numerator:} \quad \sin^2 22^\circ + \sin^2 68^\circ = \sin^2 22^\circ + \cos^2 22^\circ = 1 \quad [\text{Pythagorean identity}]$$
$$\text{Denominator:} \quad \cos^2 28^\circ + \cos^2 62^\circ = \cos^2 28^\circ + \sin^2 28^\circ = 1 \quad [\text{Pythagorean identity}]$$

$$\text{First fraction} = \frac{1}{1} = 1$$

(iii) **Simplifying the second fraction:**
Using $\sin 63^\circ = \cos 27^\circ$ and $\cos 63^\circ = \sin 27^\circ$:

$$\text{Numerator:} \quad \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ = \sin^2 63^\circ + \cos^2 63^\circ = 1$$
$$\text{Denominator:} \quad \cos 27^\circ \sin 63^\circ + \cos^2 63^\circ = \cos^2 27^\circ + \sin^2 27^\circ = 1$$

$$\text{Second fraction} = \frac{1}{1} = 1$$

Final value of E:

$$E = 1 + 1 = \boxed{2}$$

Identity primarily used: The **Pythagorean identity** $\sin^2\theta + \cos^2\theta = 1$ was the key identity, combined with the **complementary angle relationships** $\sin\theta = \cos(90^\circ - \theta)$ and $\cos\theta = \sin(90^\circ - \theta)$.

Q18. (i) **Prove:** $\dfrac{\tan 57^\circ}{\cot 33^\circ} + \dfrac{\cos 44^\circ}{\sin 46^\circ} - 3\cos 60^\circ \cdot \tan 45^\circ \cdot \csc 30^\circ = -1$

Step 1 — First term:

$$\tan 57^\circ = \tan(90^\circ - 33^\circ) = \cot 33^\circ \implies \frac{\tan 57^\circ}{\cot 33^\circ} = \frac{\cot 33^\circ}{\cot 33^\circ} = 1$$

Step 2 — Second term:

$$\cos 44^\circ = \cos(90^\circ - 46^\circ) = \sin 46^\circ \implies \frac{\cos 44^\circ}{\sin 46^\circ} = \frac{\sin 46^\circ}{\sin 46^\circ} = 1$$

Step 3 — Third term:

$$\cos 60^\circ = \frac{1}{2}, \quad \tan 45^\circ = 1, \quad \csc 30^\circ = 2$$

$$3\cos 60^\circ \cdot \tan 45^\circ \cdot \csc 30^\circ = 3 \times \frac{1}{2} \times 1 \times 2 = 3$$

LHS $= 1 + 1 - 3 = -1 =$ **RHS** ✓

(ii) **Given** $\sec A + \tan A = p$**, show** $\sec A - \tan A = \dfrac{1}{p}$ **and find** $\sin A$**:**

Using the identity $\sec^2 A - \tan^2 A = 1$:

$$\begin{aligned}(\sec A + \tan A)(\sec A - \tan A) &= 1\\ p \cdot (\sec A - \tan A) &= 1\\ \sec A - \tan A &= \frac{1}{p} \quad \checkmark\end{aligned}$$

Adding $\sec A + \tan A = p$ and $\sec A - \tan A = \dfrac{1}{p}$:

$$2\sec A = p + \frac{1}{p} = \frac{p^2+1}{p} \implies \sec A = \frac{p^2+1}{2p} \implies \cos A = \frac{2p}{p^2+1}$$

Subtracting:

$$2\tan A = p - \frac{1}{p} = \frac{p^2-1}{p} \implies \tan A = \frac{p^2-1}{2p}$$

$$\sin A = \tan A \cdot \cos A = \frac{p^2-1}{2p} \cdot \frac{2p}{p^2+1} = \boxed{\frac{p^2-1}{p^2+1}}$$